

Claim: Congruence is a symmetric relation.

Justification:

We need to show that for all integers a and b and all nonzero integers n , if $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$. So, let a and b be arbitrary integers, and let n be a nonzero integer. Assume $a \equiv b \pmod{n}$. By definition of congruence, $n \mid (a - b)$. By definition of divisibility, we can find $q \in \mathbb{Z}$ such that $nq = a - b$.

We need to show that $b \equiv a \pmod{n}$. By definition of congruence, that would mean $n \mid (b - a)$. By definition of divisibility, that would mean we need to find $Q \in \mathbb{Z}$ such that $nQ = b - a$.

Comparing what we have ($nq = a - b$) with what we need ($nQ = b - a$), we see that the only difference is the order of a and b . Thus, if we set $Q = -q$, we have

$$nQ = n(-q) = -(nq) = -(a - b) = b - a.$$

By definition of divisibility, $n \mid (b - a)$, so by definition of congruence, $b \equiv a \pmod{n}$.