

Claim: If $a \mid b_1, a \mid b_2, \dots, a \mid b_n$, then $a \mid \sum_{j=1}^n k_j b_j$ for any set of integers k_1, k_2, \dots, k_n .

Justification:

1. Assume _____.
2. We proceed by induction.
3. For the inductive base, suppose _____.
 - 3a. By _____, $a \mid b_1$.
 - 3b. By _____, we can find $q_1 \in \mathbb{Z}$ such that $aq_1 = b_1$.
 - 3c. Let k_1 be any integer.
 - 3d. By _____, $k_1 b_1 = k_1 (aq_1) = a(k_1 q_1)$.
 - 3e. By closure of integer multiplication, _____.
 - 3f. By definition, then, $a_1 \mid k_1 b_1$.
4. For the inductive hypothesis, assume _____.
5. Now suppose that $a \mid b_1, \dots, a \mid b_{n+1}$.
 - 5a. By the inductive hypothesis, _____.
 - 5b. We can rewrite $\sum_{j=1}^{n+1} k_j b_j$ as a sum of two sums, _____.
 - 5c. _____
 - 5d. By definition, then, $a_1 \mid \sum_{j=1}^{n+1} k_j b_j$.

This concludes the induction.