

**Claim:** For any ring  $R$ , the additive inverse of an element is unique.

**Justification:**

1. Let  $R$  be a ring, and let  $x \in R$ .
2. By \_\_\_\_\_,  $x$  has an additive inverse. Call it  $y$ .
3. Let  $z$  be *any* additive inverse of  $x$ .
4. By \_\_\_\_\_,  $x + y = x + z$ .
5. By \_\_\_\_\_,  $y + (x + y) = y + (x + z)$ .
6. By \_\_\_\_\_,  $(y + x) + y = (y + x) + z$ .
7. By \_\_\_\_\_,  $y + x = 0$ .
8. By \_\_\_\_\_,  $0 + y = 0 + z$ .
9. By \_\_\_\_\_,  $0 + y = y$  and  $0 + z = z$ .
10. By \_\_\_\_\_,  $y = z$ .

**Followup questions:**

1. Why is it important to consider *any* ring  $R$ ?
2. Why does the statement  $y = z$  complete the justification?
3. Which properties of a ring did you *not* need to complete this argument?