Drops in a Bucket

Common Core Standard

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of the two moving objects has greater speed.*

MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.

The Task

Your math teacher was awoken from a deep sleep very early this morning due to a loud and steady rainfall. She could not believe that she was awoken by rain, so she went outside to see what was happening. She noticed a bucket that was empty before the rain had collected a good amount of rainwater. She decided to measure the amount of water and at 4 a.m., the height of the water collected in the bucket was 63mm, at 5 a.m. the height of the water was 77mm, and at 6 a.m. the height of the water was 91mm high in the bucket. What time did the rain start?

Facilitator Notes

Note: This task was inspired by: Baltus, C. (2010, April). Connected Representations From Proportional to Linear Functions. *Mathematics Teacher, 103*(8), 591-596

1. Begin by having students make an initial prediction/estimation for the time the rain may have started. Have students compare their predictions in small groups. (Look for evidence of MP2.)
2. Have students work in small groups to complete the task. Provide access to a variety of tools for students to use as they explore the task. Tools may include chart paper, graph paper, counters, colored pencils/markers, algebra tiles, katie cubes, graphing calculator, etc. Students can record work on chart paper. (Look for evidence of MP4 and MP5.)

3. Once students have generated a solution to the task, have groups share solution and what strategies were used to solve the problem. Note: Another option is for groups to do a gallery walk of group work. (Look for evidence of MP1, MP6, and MP7.)
4. Lead class discussion about strategies, addressing misconceptions and highlighting connections between strategies. (Look for evidence of MP3.)

Follow-Up Questions

1. What strategies did you use to find you answer?
2. In what way did your group choose to represent this problem? (Look for evidence of MP4.)
3. How are the strategies the groups used to solve similar? (Look for evidence of MP2.)
4. How are the strategies the groups used to solve unique?
5. What connections did you make to the problem?

Solutions

Solution #1:

Table:

Time	**Amount of Water in the Bucket**
4 a.m.	63
5 a.m.	77
6 a.m.	91

Since: $\frac{\Delta y}{\Delta x} = \frac{77 = 63}{3 - 2} = \frac{14}{1} = 14$

Then students can work backwards from the table to determine when the rain began by subtracting 14 from the amount of water in the bucket for each past hour.

Time	**Amount of Water in the Bucket**
11 p.m.	-7
11:30 p.m.	0
12 a.m.	7
1 a.m.	21
2 a.m.	35
3 a.m.	49
4 a.m.	63
5 a.m.	77
6 a.m.	91

Students should notice that at 12 a.m. they couldn't subtract $7-14$, because they would get -7, which is impossible to have a negative value for the water collected in a bucket. Students would have to figure out that the rain must not have started on the whole hour, but on the half hour and subtracted 7 from 7 to get 0.

Solution #2

Proportions:

Some students may choose to solve this problem using proportions.

$$\frac{\Delta y}{\Delta x} = \frac{14}{1} = \frac{63}{z}$$

$$\frac{14z}{14} = \frac{63}{14}$$

$$z = 4.5$$

In this solution the 4.5 represents the number of hours ago from 4 a.m. where the initial measure was 63mm. Students should realize that if they keep working backwards an subtracting the from 4 a.m., at 11 p.m. the water in the bucket will be a -7mm, which is not possible. They have to find a time the rain began and the water amount was 0mm. This is not going to be on the whole hour. It will be on the half hour at 11:30 p.m.

Solution 3

Direct Variation

$\frac{91}{14}$ mm of water/change in the amount of water

$$\frac{91}{14} = 6.5$$

In this solution 6.5 represents the number of hours it took from when it started raining for the bucket to be at 91mm of water at 6 a.m.

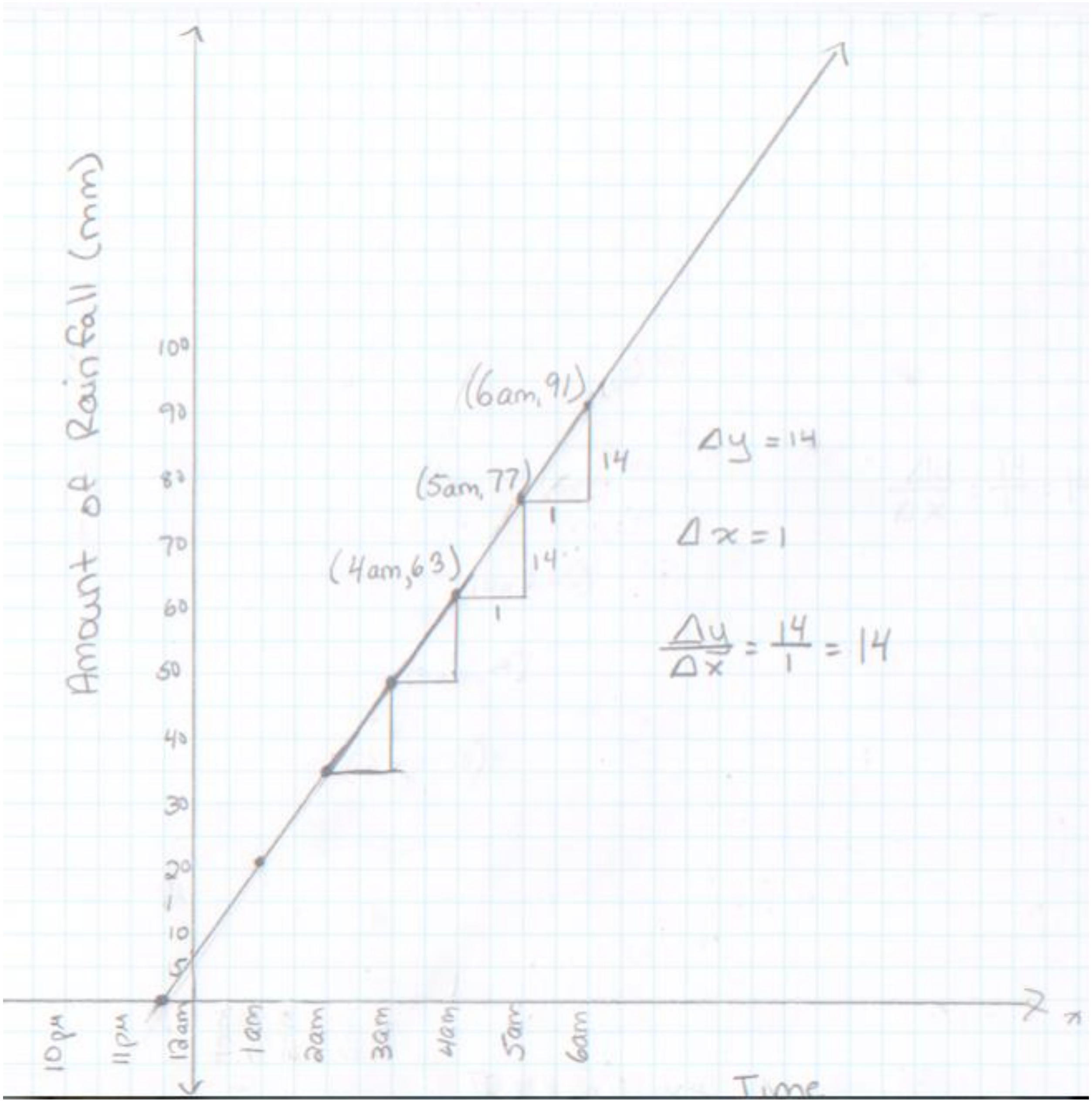
In this case, students need to subtract 6 a.m. – 6.5 hours.

6 a.m.
5 a.m.
4 a.m.
3 a.m.
2 a.m.
1 a..m.
12 a.m.
11:30 p.m.

6 a.m. – 6.5 hours gives you 11:30p.m.

Solution 4:

Graphing:


Amount of Rainfall (mm)
Time
10pm
11pm
12am
1am
2am
3am
4am
5am
6am
(4am, 63)
(5am, 77)
(6am, 91)
14
1
$\Delta y = 14$
$\Delta x = 1$
$\frac{\Delta y}{\Delta x} = \frac{14}{1} = 14$