Two-Step Equations

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

Printed: September 6, 2013

flexbook
next generation textbooks

CONCEPT 1 Two-Step Equations

Learning Objectives

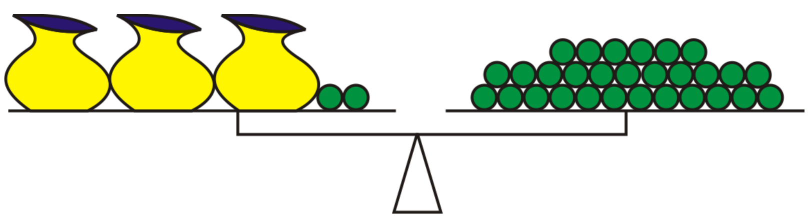
- Solve a two-step equation using addition, subtraction, multiplication, and division.
- Solve a two-step equation by combining like terms.
- Solve real-world problems using two-step equations.

Solve a Two-Step Equation

Weve seen how to solve for an unknown by isolating it on one side of an equation and then evaluating the other side. Now well see how to solve equations where the variable takes more than one step to isolate.

Example 1

Rebecca has three bags containing the same number of marbles, plus two marbles left over. She places them on one side of a balance. Chris, who has more marbles than Rebecca, adds marbles to the other side of the balance. He finds that with 29 marbles, the scales balance. How many marbles are in each bag? Assume the bags weigh nothing.



Solution

We know that the system balances, so the weights on each side must be equal. If we use x to represent the number of marbles in each bag, then we can see that on the left side of the scale we have three bags (each containing x marbles) plus two extra marbles, and on the right side of the scale we have 29 marbles. The balancing of the scales is similar to the balancing of the following equation.

$$3x+2=29$$

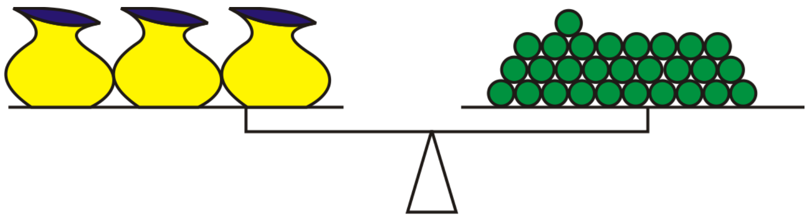
Three bags plus two marbles **equals** 29 marbles

To solve for x, we need to first get all the variables (terms containing an x) alone on one side of the equation. Weve already got all the xs on one side; now we just need to isolate them.

$$\begin{aligned} 3x+2&=29 \\ 3x+2-2&=29-2 && \text{Get rid of the 2 on the left by subtracting it from both sides.} \\ 3x&=27 \\ \frac{3x}{3}&=\frac{27}{3} && \text{Divide both sides by 3.} \\ x&=9 \end{aligned}$$

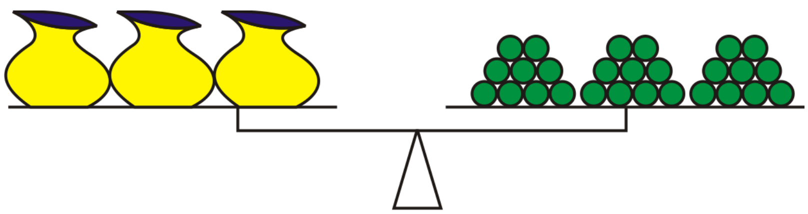
There are nine marbles in each bag.

We can do the same with the real objects as we did with the equation. Just as we subtracted 2 from both sides of the equals sign, we could remove two marbles from each side of the scale. Because we removed the same number of marbles from each side, we know the scales will still balance.



Then, because there are three bags of marbles on the left-hand side of the scale, we can divide the marbles on the right-hand side into three equal piles. You can see that there are nine marbles in each.

Three bags of marbles ***balances*** *three piles of nine marbles.*



So each bag of marbles balances nine marbles, meaning that each bag contains nine marbles.

Check out http://www.mste.uiuc.edu/pavel/java/balance/ for more interactive balance beam activities!

Example 2

Solve $6(x+4) = 12$.

This equation has the x buried in parentheses. To dig it out, we can proceed in one of two ways: we can either distribute the six on the left, or divide both sides by six to remove it from the left. Since the right-hand side of the equation is a multiple of six, it makes sense to divide. That gives us $x+4 = 2$. Then we can subtract 4 from both sides to get $x = -2$.

Example 3

Solve $\frac{x-3}{5} = 7$.

Its always a good idea to get rid of fractions first. Multiplying both sides by 5 gives us $x-3 = 35$, and then we can add 3 to both sides to get $x = 38$.

Example 4

Solve $\frac{5}{9}(x+1) = \frac{2}{7}$.

First, well cancel the fraction on the left by multiplying by the reciprocal (the multiplicative inverse).

$$\frac{9}{5} \cdot \frac{5}{9}(x+1) = \frac{9}{5} \cdot \frac{2}{7}$$
$$(x+1) = \frac{18}{35}$$

Then we subtract 1 from both sides. ($\frac{35}{35}$ is equivalent to 1.)

$$\begin{aligned} x+1 &= \frac{18}{35} \\ x+1-1 &= \frac{18}{35}-\frac{35}{35} \\ x &= \frac{18-35}{35} \\ x &= \frac{-17}{35} \end{aligned}$$

These examples are called **two-step equations**, because we need to perform two separate operations on the equation to isolate the variable.

Solve a Two-Step Equation by Combining Like Terms

When we look at a linear equation we see two kinds of terms: those that contain the unknown variable, and those that dont. When we look at an equation that has an x on both sides, we know that in order to solve it, we need to get all the $x-$terms on one side of the equation. This is called **combining like terms**. The terms with an x in them are **like terms** because they contain the same variable (or, as you will see in later chapters, the same combination of variables).

TABLE 1.1:

Like Terms	**Unlike Terms**
$4x, 10x, -3.5x$, and $\frac{x}{12}$	$3x$ and $3y$
$3y, 0.000001y$, and y	$4xy$ and $4x$
$xy, 6xy$, and $2.39xy$	$0.5x$ and 0.5

To add or subtract like terms, we can use the Distributive Property of Multiplication.

$$\begin{aligned} 3x+4x &= (3+4)x = 7x \\ 0.03xy-0.01xy &= (0.03-0.01)xy = 0.02xy \\ -y+16y+5y &= (-1+16+5)y = 10y \\ 5z+2z-7z &= (5+2-7)z = 0z = 0 \end{aligned}$$

To solve an equation with two or more like terms, we need to combine the terms first.

Example 5

Solve $(x+5)-(2x-3)=6$.

There are two like terms: the x and the $-2x$ (dont forget that the negative sign applies to everything in the parentheses). So we need to get those terms together. The associative and distributive properties let us rewrite the equation as $x+5-2x+3=6$, and then the commutative property lets us switch around the terms to get $x-2x+5+3=6$, or $(x-2x)+(5+3)=6$.

$(x-2x)$ is the same as $(1-2)x$, or $-x$, so our equation becomes $-x+8=6$

Subtracting 8 from both sides gives us $-x=-2$.

And finally, multiplying both sides by -1 gives us $x=2$.

Example 6

Solve $\frac{x}{2} - \frac{x}{3} = 6$.

This problem requires us to deal with fractions. We need to write all the terms on the left over a common denominator of six.

$$\frac{3x}{6} - \frac{2x}{6} = 6$$

Then we subtract the fractions to get $\frac{x}{6} = 6$.

Finally we multiply both sides by 6 to get $x = 36$.

Solve Real-World Problems Using Two-Step Equations

The hardest part of solving word problems is translating from words to an equation. First, you need to look to see what the equation is asking. What is the **unknown** for which you have to solve? That will be what your **variable** stands for. Then, follow what is going on with your variable all the way through the problem.

Example 7

An emergency plumber charges $65 as a call-out fee plus an additional $75 per hour. He arrives at a house at 9:30 and works to repair a water tank. If the total repair bill is $196.25, at what time was the repair completed?

In order to solve this problem, we collect the information from the text and convert it to an equation.

Unknown: time taken in hours this will be our x

The bill is made up of two parts: a call out fee and a per-hour fee. The call out is a flat fee, and independent of $x$its the same no matter how many hours the plumber works. The per-hour part depends on the number of hours (x). So the total fee is $65 (no matter what) plus 75x$ (where x is the number of hours), or $65 + 75x$.

Looking at the problem again, we also can see that the total bill is $196.25. So our final equation is $196.25 = 65 + 75x$.

Solving for x:

$$\begin{aligned} 196.25 &= 65 + 75x && \text{Subtract 65 from both sides.} \\ 131.25 &= 75x && \text{Divide both sides by 75.} \\ 1.75 &= x && \text{The job took 1.75 hours.} \end{aligned}$$

Solution

The repair job was completed 1.75 hours after 9:30, so it was completed at 11:15AM.

Example 8

When Asia was young her Daddy marked her height on the door frame every month. Asias Daddy noticed that between the ages of one and three, he could predict her height (in inches) by taking her age in months, adding 75 inches and multiplying the result by one-third. Use this information to answer the following:

a) *Write an equation linking her predicted height, h, with her age in months, m.*

b) *Determine her predicted height on her second birthday.*

c) *Determine at what age she is predicted to reach three feet tall.*

Solution

a) To convert the text to an equation, first determine the type of equation we have. We are going to have an equation that links **two variables**. Our unknown will change, depending on the information we are given. For example, we could solve for height given age, or solve for age given height. However, the text gives us a way to determine **height**. Our equation will start with $h =$.

The text tells us that we can predict her height by taking her age in months, adding 75, and multiplying by $\frac{1}{3}$. So our equation is $h = (m+75) \cdot \frac{1}{3}$, or $h = \frac{1}{3}(m+75)$.

b) To predict Asias height on her second birthday, we substitute $m = 24$ into our equation (because 2 years is 24 months) and solve for h.

$$
\begin{aligned}
h &= \frac{1}{3}(24+75) \\
h &= \frac{1}{3}(99) \\
h &= 33
\end{aligned}
$$

Asias height on her second birthday was predicted to be 33 inches.

c) To determine the predicted age when she reached three feet, substitute $h = 36$ into the equation and solve for m.

$$
\begin{aligned}
36 &= \frac{1}{3}(m+75) \\
108 &= m+75 \\
33 &= m
\end{aligned}
$$

Asia was predicted to be 33 months old when her height was three feet.

Example 9

To convert temperatures in Fahrenheit to temperatures in Celsius, follow the following steps: Take the temperature in degrees Fahrenheit and subtract 32. Then divide the result by 1.8 and this gives the temperature in degrees Celsius.

a) *Write an equation that shows the conversion process.*

b) *Convert 50 degrees Fahrenheit to degrees Celsius.*

c) *Convert 25 degrees Celsius to degrees Fahrenheit.*

d) *Convert -40 degrees Celsius to degrees Fahrenheit.*

a) The text gives the process to convert Fahrenheit to Celsius. We can write an equation using two variables. We will use f for temperature in Fahrenheit, and c for temperature in Celsius.

First we take the temperature in Fahrenheit and subtract 32. $f-32$

Then divide by 1.8. $\frac{f-32}{1.8}$

This equals the temperature in Celsius. $c=\frac{f-32}{1.8}$

In order to convert from one temperature scale to another, simply substitute in for whichever temperature you know, and solve for the one you dont know.

b) To convert 50 degrees Fahrenheit to degrees Celsius, substitute $f=50$ into the equation.

$$c=\frac{50-32}{1.8}$$
$$c=\frac{18}{1.8}$$
$$c=10$$

50 degrees Fahrenheit is equal to 10 degrees Celsius.

c) To convert 25 degrees Celsius to degrees Fahrenheit, substitute $c=25$ into the equation:

$$25=\frac{f-32}{1.8}$$
$$45=f-32$$
$$77=f$$

25 degrees Celsius is equal to 77 degrees Fahrenheit.

d) To convert -40 degrees Celsius to degrees Fahrenheit, substitute $c=-40$ into the equation.

$$-40=\frac{f-32}{1.8}$$
$$-72=f-32$$
$$-40=f$$

-40 degrees Celsius is equal to -40 degrees Fahrenheit. (No, thats not a mistake! This is the one temperature where they are equal.)

Lesson Summary

- Some equations require more than one operation to solve. Generally it, is good to go from the outside in. If there are parentheses around an expression with a variable in it, cancel what is outside the parentheses first.
- Terms with the same variable in them (or no variable in them) are **like terms. Combine like terms** (adding or subtracting them from each other) to simplify the expression and solve for the unknown.

Review Questions

1. Solve the following equations for the unknown variable.
   (a) $1.3x - 0.7x = 12$
   (b) $6x - 1.3 = 3.2$
   (c) $5x - (3x + 2) = 1$
   (d) $4(x + 3) = 1$
   (e) $5q - 7 = \frac{2}{3}$
   (f) $\frac{3}{5}x + \frac{5}{2} = \frac{2}{3}$
   (g) $s - \frac{3s}{8} = \frac{5}{6}$
   (h) $0.1y + 11 = 0$
   (i) $\frac{5q-7}{12} = \frac{2}{3}$
   (j) $\frac{5(q-7)}{12} = \frac{2}{3}$
   (k) $33t - 99 = 0$
   (l) $5p - 2 = 32$
   (m) $10y + 5 = 10$
   (n) $10(y + 5) = 10$
   (o) $10y + 5y = 10$
   (p) $10(y + 5y) = 10$
2. Jade is stranded downtown with only $10 to get home. Taxis cost $0.75 per mile, but there is an additional $2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money.
3. Jasmins Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs $150 for the afternoon, and the food will cost $3 per person. Andrew, Jasmins Dad, has a budget of $300. Write an equation and use it to determine the maximum number of guests he can invite.
4. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.
   (a) If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?
   (b) How many visits can he make if he does not?
   (c) If he increases his budget to $160, how many visits can he make as a member?
   (d) And how many as a non-member?
5. For an upcoming school field trip, there must be one adult supervisor for every five children.
   (a) If the bus seats 40 people, how many children can go on the trip?
   (b) How many children can go if a second 40-person bus is added?
   (c) Four of the adult chaperones decide to arrive separately by car. Now how many children can go in the two buses?