

Lesson 55

Perimeter Problems with Related Variables

Take your skill at word problems to a new level in this section. All the problems are the same type, so that you can focus on the extremely useful and important algebraic technique involved.

Comparing Solutions

The formula for the perimeter of a rectangle is pretty familiar by now: $P = 2(L + W)$ or $P = 2L + 2W$.

Compare the two problems below:

A rectangle has **length 10 inches**.
The **width is 5 inches more than the length**.
Find the perimeter of the rectangle.

$$P = 2L + 2W$$

$$P = 2(10) + 2(10 + 5)$$

A rectangle has **length L inches**.
The **width is 5 inches more than the length**.
The perimeter of the rectangle is **50 inches**.
Find the length and width.

$$P = 2L + 2W$$

$$50 = 2(L) + 2(L + 5)$$

In the first problem, we know the length is 10 inches, so to find the width, we just add 5 inches to 10 inches. In the second problem, we don't know the length, so we have to represent it with a variable, L . To find the width, we add 5 to the length, but since the length is written as a variable rather than a number, we have to write $L + 5$ for the width.

Both the first and the second version of the problem can be solved, because in both we end up with an equation with only one variable.

$$P = 2(10) + 2(10 + 5)$$

$$P = 20 + 30 = 50$$

The perimeter is 50 inches.

$$50 = 2(L) + 2(L + 5)$$

Use the distributive property to simplify:

$$50 = 2L + (2L + 10)$$

Combine like terms: $50 = 4L + 10$

Solve: $4L + 10 = 50$ $4L + 10 - 10 = 50 - 10$

$$4L = 40$$
 $4L / 4 = 40 / 4$

$$L = 10$$

The length is 10 inches.

The instructions were to find the length *and* width, so there is one more step. The width is 5 more than the length, so

$$\text{The width is 15 inches.}$$

The technique of using a relationship between two variables to write an equation is an important algebraic skill. The perimeter problems used in demonstration here are not very practical, but the technique is.

Writing the Equation

There are four steps to solving problems like the second example. Only the first step is new.

1. Substitute the values you know and the relationship given between the length and width in the perimeter formula.
2. Simplify the equation. You may have to use the distributive property, or multiply expressions, or combine like terms, or all three.
3. Solve the equation and find the value of the variable.
4. Use the value you found to find the other, related variable.

Example A rectangle has width 2 inches more than the length. The perimeter is 48 inches. Find the length and width of the rectangle.

<p>Step 1</p> <p>Substitute the values you know and the relationship given between the length and width in the perimeter formula.</p> <p>The length is unknown, so write the length as L. The width is 2 inches more than L, so the width is L + 2. The perimeter, P, is 48 inches.</p> $P = 2L + 2W$ $48 = 2L + 2(L + 2)$ <p>Note the key substitution, shown in red above. We don't know either L or W, but we know W is 2 more than L, so we can write the equation with only a single variable.</p>	<p>Step 2</p> <p>Simplify the equation.</p> $48 = 2L + 2(L + 2)$ <p>Use the distributive property:</p> $48 = 2L + 2L + 4$ <p>Next, combine like terms:</p> $48 = 2L + 2L + 4$ $48 = 4L + 4$ <p>You can switch around the sides of the equation if you prefer that format:</p> $4L + 4 = 48$
<p>Step 3</p> <p>Solve the equation and find the value of the variable.</p> $4L + 4 = 48$ $4L + 4 - 4 = 48 - 4$ $4L = 44$ $4L / 4 = 44 / 4$ $L = 11$ <p>The length is 11 inches.</p>	<p>Step 4</p> <p>Use the value you found to find the other, related variable.</p> <p>The width is 2 inches more than the length. The length is 11 inches. Therefore, the width is $11 + 2 = 13$ inches. The width is 13 inches.</p>

Let's check our work above and make sure that the perimeter is really 48 inches:

$$P = 2L + 2W = 2(11 \text{ in.}) + 2(13 \text{ in.}) = 22 \text{ in.} + 26 \text{ in.} = 48 \text{ in.}$$

This confirms that the solution is correct.

The New Substitution Technique

The new algebraic technique here is writing one variable in terms of another, so that the equation will have only one variable. Writing the mathematical relationship between the variables can sometimes feel tricky. One helpful technique is to pretend that you know the missing number and see what you would do in a purely arithmetic situation.

For example, suppose the problem specifies that “The width is half the length,” and you’re not sure how to write that with variables. Pretend that you know the length, and make up any number you like. If the length is 10 inches, for example, and the width is half the length, can you figure out the width? You know that the width is 5 inches, and you found it by dividing 10 by 2. So if the length is represented by L , you do the same thing and divide L by 2. We would write the width as $L/2$.

Let’s practice only Step 1 in the next few examples.

Example Write the equation for the problem using the perimeter formula. In each case the perimeter is 60 inches.

<p>The width of a rectangle is four more than the length. Width is four more than length, so W is $L + 4$</p> $P = 2L + 2W$ $60 = 2L + 2(L + 4)$	<p>The width of a rectangle is four less than the length. Width is four less than length, so W is $L - 4$</p> $P = 2L + 2W$ $60 = 2L + 2(L - 4)$
<p>The width of a rectangle is four times the length. Width is four times the length, so W is $4L$</p> $P = 2L + 2W$ $60 = 2L + 2(4L)$	<p>The width of a rectangle is one-fourth the length. Width is one-fourth length, so W is $L / 4$</p> $P = 2L + 2W$ $60 = 2L + 2(L / 4)$

Now let’s put it all together again.

Example A rectangle has width three times the length. The perimeter is 56 inches. Find the length and width.

<p>Step 1</p> <p>The length is unknown, so length is L. The width is three times the length, so the width is $3L$. The perimeter, P, is 56 inches.</p> $P = 2L + 2W$ $56 = 2L + 2(3L)$	<p>Step 2</p> $56 = 2L + 2(3L)$ $56 = 2L + 6L$ $56 = 8L$ <p>Step 3</p> $8L = 56 \quad 8L / 8 = 56 / 8$ $L = 7$ <p>Step 4</p> <p>The length is 7 inches. The width is three times the length. Width is $3(7 \text{ in.}) = 21 \text{ in.}$ The width is 21 inches.</p>
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Check: A rectangle with length 7 inches and width 21 inches has perimeter $2(7) + 2(21) = 14 + 42 = 56$ inches.

One last example.

Example A rectangle has width one-half the length. The perimeter is 72 inches. Find the length and width.

Step 1

The length is unknown, so length is **L**.
The width is three times the length, so the width is **L/2**.
The perimeter, **P**, is 100 inches.

$$P = 2L + 2W$$
$$72 = 2L + 2(L/2)$$

Step 2

$$72 = 2L + 2(L/2)$$
$$72 = 2L + L$$
$$72 = 3L$$

Step 3

$$3L = 72 \quad 3L / 3 = 72 / 3$$
$$L = 24$$

Step 4

The length is 24 inches.
The width is half the length.
Width is $24/2 = 12$ in.
The width is 12 inches.

Check: A rectangle with length 24 inches and width 12 inches has perimeter $2(24) + 2(12) = 48 + 24 = 72$ inches.



Lesson 55: Perimeter Problems with Related Variables

Worksheet

Name _____

Solving Perimeter Problems with Related Variables

1. Substitute the values you know and the relationship given between the length and width in the perimeter formula.
2. Simplify the equation. You may have to use the distributive property, or multiply expressions, or combine like terms, or all three.
3. Solve the equation and find the value of the variable.
4. Use the value you found to find the other, related variable.

Practice Step 1 by filling in the equation. The perimeter of each rectangle is 120 inches.

1. The width of the rectangle is 10 more than the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$	2. The width of the rectangle is 10 less than the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$
3. The width of the rectangles is 5 times the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$	4. The width of the rectangle is 1/5 of the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$
5. The width of the rectangle is 3 less than the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$	6. The width of the rectangle is 1.5 times the length. $P = 2L + 2W$ $120 = 2L + 2(\quad)$

Solve to find the length and width of the rectangle.

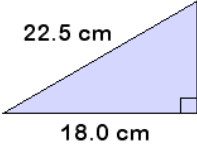
7. The perimeter is 82 inches, and the width is 3 inches less than the length.	8. The perimeter is 100 inches, and the width is 1.5 times the length.
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Lesson 55: Perimeter Problems with Related Variables

Homework 55A

Name _____

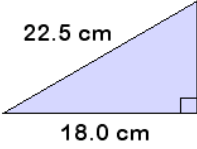
1. Use the distributive property to simplify $2(6n - 5)$.	2. Combine like terms to simplify $3k + 9k + 11$.
3. Simplify $4(8b + 12) - 5(2b)$.	4. Simplify $7(-3w) + 2(5w - 4)$.
5. Write equivalent fractions with a common denominator. $\frac{4}{7b}$ $\frac{3}{14}$	6. Add $\frac{4}{7b} + \frac{3}{14}$.
7. Write equivalent fractions with a common denominator. $\frac{5}{a}$ $\frac{2}{a^2}$	8. Subtract $\frac{5}{a} - \frac{2}{a^2}$.
9. Evaluate a. $\sqrt{8^2 - 9^2}$ b. $\sqrt{(8 - 9)^2}$ c. $\sqrt{4^2 \cdot 5^2}$ d. $\sqrt{4^2} \cdot \sqrt{5^2}$	10. Evaluate a. $\frac{8 + 3(-5)}{14(-5)}$ b. $\frac{4(5) - 2}{5^2}$ c. $\frac{\sqrt{1}}{\sqrt{5^2 - 3^2}}$

<p>11. Find the length of the side not marked. Round to the nearest tenth if rounding is necessary.</p> 	<p>12. If the distance from the wall is 1 foot for every 4 feet of ladder length, how far up the wall will a 12 foot ladder reach?</p>
<p>13. Match the definitions with the terms.</p> <p>natural numbers whole numbers integers rational numbers irrational numbers real numbers</p> <p>a. _____: All the numbers on the number line.</p> <p>b. _____: The quotients of integers, excepting division by zero.</p> <p>c. _____: The counting numbers.</p> <p>d. _____: Positive and negative whole numbers.</p>	<p>14. Fill in the expression for the width. The perimeter of each rectangle is 80 inches. You don't have to solve the equations.</p> <p>a. The width is 6 more than the length. $80 = 2L + 2(\quad)$</p> <p>b. The width is $\frac{1}{6}$ of the length. $80 = 2L + 2(\quad)$</p> <p>c. The width 6 times the length. $80 = 2L + 2(\quad)$</p> <p>d. The width is 6 less than the length. $80 = 2L + 2(\quad)$</p>
<p>15. The width of a rectangle is 4 less than the length. The perimeter is 40 inches. Find the length and width.</p>	<p>16. A rectangle has width 2.5 times its length. The perimeter is 70 inches. What are the length and width?</p>

Lesson 55: Perimeter Problems with Related Variables

Homework 55A Answers

<p>1. Use the distributive property to simplify $2(6n - 5)$.</p> $12n - 10$	<p>2. Combine like terms to simplify $3k + 9k + 11$.</p> $12k + 11$
<p>3. Simplify $4(8b + 12) - 5(2b)$.</p> $32b + 48 - 10b$ $22b + 48$	<p>4. Simplify $7(-3w) + 2(5w - 4)$.</p> $-21w + 10w - 8$ $-11w - 8$
<p>5. Write equivalent fractions with a common denominator.</p> $\frac{4}{7b} = \frac{4}{7 \cdot b} \cdot \frac{2}{2} = \frac{8}{14b}$ $\frac{3}{14} = \frac{3}{2 \cdot 7} \cdot \frac{b}{b} = \frac{3b}{14b}$	<p>6. Add $\frac{4}{7b} + \frac{3}{14}$.</p> $= \frac{8}{14b} + \frac{3b}{14b} = \frac{8 + 3b}{14b}$
<p>7. Write equivalent fractions with a common denominator.</p> $\frac{5}{a} = \frac{4}{a} \cdot \frac{a}{a} = \frac{4a}{a^2}$ $\frac{2}{a^2} = \frac{2}{a \cdot a} = \frac{2}{a^2}$	<p>8. Subtract $\frac{5}{a} - \frac{2}{a^2}$.</p> $= \frac{4a}{a^2} - \frac{2}{a^2} = \frac{4a - 2}{a^2}$
<p>9. Evaluate</p> <p>a. $\sqrt{8^2 - 9^2} = \sqrt{64 - 81} = \sqrt{-17}$ not a real number</p> <p>b. $\sqrt{(8 - 9)^2} = \sqrt{(-1)^2} = \sqrt{1} = 1$</p> <p>c. $\sqrt{4^2 \cdot 5^2} = \sqrt{16 \cdot 25} = \sqrt{400} = 20$</p> <p>d. $\sqrt{4^2} \cdot \sqrt{5^2} = \sqrt{16} \cdot \sqrt{25} = 4 \cdot 5 = 20$</p>	<p>10. Evaluate</p> <p>a. $\frac{8 + 3(-5)}{14(-5)} = \frac{8 - 15}{-70} = \frac{-7}{-70} = \frac{1}{10}$</p> <p>b. $\frac{4(5) - 2}{5^2} = \frac{20 - 2}{25} = \frac{18}{25}$</p> <p>c. $\frac{\sqrt{1}}{\sqrt{5^2 - 3^2}} = \frac{1}{\sqrt{25 - 9}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$</p>

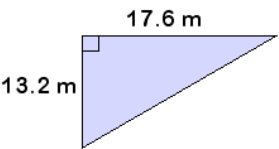
<p>11. Find the length of the side not marked. Round to the nearest tenth if rounding is necessary.</p>  <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> $a^2 + b^2 = c^2$ $a^2 + 18^2 = 22.5^2$ $a^2 + 324 = 506.25$ $a^2 = 506.25 - 324 = 182.25$ $a = \sqrt{182.25} = 13.5$ </div> <p style="text-align: center;">The side is 13.5 cm.</p>	<p>12. If the distance from the wall is 1 foot for every 4 feet of ladder length, how far up the wall will a 12 foot ladder reach?</p> <p style="text-align: center;">The ladder is 12 feet, so the distance from the wall is $12 / 4 = 3$ feet.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> $a^2 + 3^2 = 12^2$ $a^2 + 9 = 144$ $a^2 = 144 - 9 = 135$ $a = \sqrt{135} \approx 11.619$ </div> <p style="text-align: center;">The ladder reaches about 11.6 feet up the wall.</p>
<p>13. Match the definitions with the terms.</p> <p>natural numbers whole numbers integers rational numbers irrational numbers real numbers</p> <p>a. ___ real numbers ___: All the numbers on the number line.</p> <p>b. _ rational numbers ___: The quotients of integers, excepting division by zero.</p> <p>c. ___ natural numbers ___: The counting numbers.</p> <p>d. _____ integers _____: Positive and negative whole numbers.</p>	<p>14. Fill in the expression for the width. The perimeter of each rectangle is 80 inches. You don't have to solve the equations.</p> <p>a. The width is 6 more than the length. $80 = 2L + 2(L + 6)$</p> <p>b. The width is 1/6 of the length. $80 = 2L + 2(L / 6)$</p> <p>c. The width 6 times the length. $80 = 2L + 2(6L)$</p> <p>d. The width is 6 less than the length. $80 = 2L + 2(L - 6)$</p>
<p>15. The width of a rectangle is 4 less than the length. The perimeter is 40 inches. Find the length and width.</p> $P = 2L + 2W$ $40 = 2L + 2(L - 4)$ $40 = 2L + 2L - 8$ $40 = 4L - 8$ $4L - 8 = 40 \quad 4L - 8 + 8 = 40 + 8$ $4L = 48 \quad 4L / 4 = 48 / 4$ $L = 12$ <p style="text-align: center;">The length is 12 inches. The width is $12 - 4 = 8$ inches.</p>	<p>16. A rectangle has width 2.5 times its length. The perimeter is 70 inches. What are the length and width?</p> $P = 2L + 2W$ $70 = 2L + 2(2.5L)$ $70 = 2L + 5L$ $7L = 70 \quad 7L / 7 = 70 / 7$ $L = 10$ <p style="text-align: center;">The length is 10 inches. The width is $(2.5)(10) = 25$ inches.</p>

Lesson 55: Perimeter Problems with Related Variables

Homework 55B

Name _____

1. Use the distributive property to simplify $4(3g + 8)$.	2. Combine like terms to simplify $8y - 5 - 9y$.
3. Simplify $2(5t - 7) - 4(-2t)$.	4. Simplify $6(m - 3) - 2(3m - 5)$.
5. Write equivalent fractions with a common denominator. $\frac{8}{3x}$ $\frac{7}{6x}$	6. Add $\frac{8}{3x} + \frac{7}{6x}$.
7. Write equivalent fractions with a common denominator. $\frac{5}{2y}$ $\frac{3}{8}$	8. Subtract $\frac{5}{2y} - \frac{3}{8}$.
9. Evaluate a. $(\sqrt{6})^2$ b. $\sqrt{6^2}$ c. $\sqrt{-6^2}$ d. $\sqrt{(-6)^2}$	10. Evaluate a. $\frac{20 - 3(7)}{8(7)}$ b. $\frac{8(-5) + 7(-5)}{6(-5)}$ c. $\frac{\sqrt{9}}{\sqrt{16}} - \sqrt{\frac{9}{16}}$

<p>11. Find the length of the side not marked. Round to the nearest tenth if rounding is necessary.</p> 	<p>12. If the distance from the wall is 1 foot for every 4 feet of ladder length, how far up the wall will a 16 foot ladder reach?</p>
<p>13. Give an example of</p> <p>a. a number that is rational, but is not an integer.</p> <p>b. a number that is irrational.</p> <p>c. a number that is an integer, but not a whole number.</p> <p>d. a number that is a natural number, but not a whole number.</p>	<p>14. Fill in the expression for the width. The perimeter of each rectangle is 80 inches. You don't have to solve the equations.</p> <p>a. The width is 1 less than the length. $80 = 2L + 2(\quad)$</p> <p>b. The width is 4 times the length. $80 = 2L + 2(\quad)$</p> <p>c. The width is $\frac{2}{3}$ of the length. $80 = 2L + 2(\quad)$</p> <p>d. The width is 1 more than the length. $80 = 2L + 2(\quad)$</p>
<p>15. The width of a rectangle is 7 more than the length. The perimeter is 66 inches. Find the length and width.</p>	<p>16. A rectangle has width 8 times its length. The perimeter is 90 inches. What are the length and width?</p>