Multiplying Integers

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CONCEPT 1 Multiplying Integers

Introduction

Welcome to Jafakids

Tyler's pen pal Amelia is a member of Jafakids in Auckland New Zealand. Tyler loves mountain biking, particularly in upstate New York in the summer. Through his letters with Amelia, he has discovered that Amelia is a member of a mountain biking group that meets daily in Auckland.

Amelia told him all about it in her last letter. She told him that Jafakids is a club that was started to promote fitness. You have to be 11 years old to join and everyday from 5:30 to 7:30 pm kids get a chance to go mountain biking. Some of the groups ride on some very difficult single track trails. Some of the kids decide to race and compete. This is Amelia's first year and so she is still learning how to tackle some of the most challenging terrain.

"I ride a lot and love it," Amelia wrote to Tyler. "Last week we were at a big park called Wood Hill Bike Park. It is over 100 kilometers long, and there are some difficult trails to ride. We went up to the top of this big hill. Our instructors told us that for every kilometer in altitude that we climbed, we would drop about 2° in temperature. We ended up traveling 5 vertical kilometers."

Tyler stopped reading the letter. He was puzzled. If the group had traveled 5 vertical kilometers, and it was 70° when they started, what was the temperature when they stopped?

To figure this problem out, you will need to understand how to multiply integers. Pay close attention to this lesson and you will understand how to figure out the answer to Tyler's question.

What You Will Learn

In this lesson, you will learn the following skills:

- Find products of positive integers.
- Find products of positive and negative integers.
- Find products of negative integers.
- Evaluate numerical and algebraic expressions involving integer multiplication.

Teaching Time

I. **Find Products of Positive Integers**

This lesson focuses on finding the **products** of integers.

What is a product?

A **product** is the answer of a multiplication problem. When we talk about products we are talking about multiplying integers. Remember that an **integer** is the set of whole numbers and their opposites.

Let's look at finding the products of positive integers. What does it mean to multiply?

Remember that multiplication is repeated addition. When a value is being multiplied repeatedly, we can say that multiplication is a short cut for this repeated addition.

Example

3(4) = ____

This means 3 times 4. A set of parentheses is a way to show multiplication. This means that four is being added three times.

4 + 4 + 4 = 12

The answer is 12.

We can find the product of any two positive integers just as we would multiply any two positive whole numbers.

Example

5(7) = ____

7 + 7 + 7 + 7 + 7 = 35

The answer is 35.

These two examples point us to a rule.

Use what you have just learned to multiply the following positive integers.

1. **8(3) =** ____
2. **5(9) =** ____
3. **2(7) =** ____

Take a few minutes to check your work with a partner.

II. **Find Products of Positive and Negative Integers**

We can also find products of positive and negative integers.

What does it mean to multiply a positive and a negative integer?

When we multiply a positive and a negative integer, it means that the negative number is being added so many times. Once again, we have an example of repeated addition. Let's look at an example.

Example

6(-5) = ____

Here negative five is being added six times.

-5 + -5 + -5 + -5 + -5 + -5 = -30

The answer is negative thirty.

Example

2(-3) = ____

Here negative three is being added twice. That is the repeated addition.

-3 + -3 = -6

The answer is -6.

We can write a rule for multiplying positive and negative integers based on these two examples.

Does it matter if we write the positive number first or the negative number first?

No. It doesn't matter because of the Commutative Property of Multiplication. You can check out a review of this property in the Vocabulary Review at the end of the lesson.

Practice a few of these on your own.

1. **3(-7) =** ____
2. **5(-9) =** ____
3. **4(-3) =** ____

Check your work with a friend. Is your work accurate?

III. **Find Products of Negative Integers**

We can also find the product of two negative integers.

Let's say that we wanted to multiply the following two negative integers.

(-5)(-2) = ____

We know that 5 times 2 is 10, but we don't know what sign to use. Is the answer positive or negative?

Well, what happens if we multiply -5 and 0?

-5(0) = 0

The answer is zero.

What happens if we multiply -5 by 1?

-5(1) = -5

The answer is negative five. We already know that a negative times a positive is a negative.

What happens if we multiply -5 by 2?

-5(2) = -10

Notice that each time we multiply by a new digit in order, 0, 1, 2, we have a difference of five in our answer.

Here are the three products we just arrived at. These were the products of multiplying by 0, 1 and 2.

0, -5, -10

We can say that if we multiply by -1 and -2, our answer will go 5 the other way. These moved negative. If we multiply by the opposite sign, the values should move positive.

$$-5(0) = 0$$
$$-5(-1) = 5$$
$$-5(-2) = 10$$

Here is the rule for multiplying two negative integers.

Practice a few of these on your own.

1. **-9(-8) = ____**
2. **(-4)(-12) = ____**
3. **(-5)(-13) = ____**

Take a few minutes to check your answers with a friend. Did you remember the rule about the sign?

IV. **Evaluate Numerical and Algebraic Expressions Involving Integer Multiplication**

Now that you have learned how to multiply integers, you are ready to put this into practice when evaluating **numerical** and **algebraic expressions.**

You will need the rules for multiplying integers to evaluate expressions.

Positive $\times$positive = positive

Negative $\times$positive = negative

Negative $\times$negative = positive

Let's begin by evaluating numerical expressions.

A numerical expression is an expression that has multiple numbers and sometimes multiple operations. To evaluate a numerical expression involving integers, you will need to remember all of your integer rules and apply them as you work.

Example

(-3)(-5) + (-2)(7)

In this example, we have two operations, multiplication and addition.

Let's begin with the multiplication. Remember that whenever you see two sets of parentheses next to each other that it means that we need to multiply. Let's multiply each part of the expression.

$$(-3)(-5) = 15$$
$$(-2)(7) = -14$$

Now we can add these products.

15 + -14 = 1

Our final answer is 1.

Numerical expressions can take many different forms too. The key is that there are often more than two numbers and at least two operations. Let's look at another example.

Example

-2(-5 + -3 + 6)

Here we need to find a sum inside of the parentheses and then multiply this sum by negative two.

$$-5+-3+6 = -8+6 = -2$$
$$-2(-2) = 4$$

The answer is 4.

We can also evaluate algebraic expressions. What is an algebraic expression?

An algebraic expression uses a combination of numbers, operations and variables. When you evaluate an algebraic expression you are often given a value for the variable. You substitute this value into the expression for the variable and then evaluate the expression.

Let's look at an example.

Example

$-4(10x)$ *when* $x = 3$

To evaluate this expression, we have to substitute the given value of x into the expression. Remember that a number next to a variable means multiply.

$$-4(10 \times 3)$$
$$-4(30)$$

Next, we multiply negative four by thirty.

-4(30) = -120

The answer is -120.

Practice what you have learned by evaluating the following expressions.

1. -6(-1 + 5)
2. $7(3y)$ *when* $y = -2$
3. -3(2) – (-2)(-4)

Take a few minutes to check your work with a peer.

Real Life Example Completed

Welcome to Jafakids

Here is the original problem once again. Reread the problem and underline any important information.

Tyler's pen pal Amelia is a member of Jafakids in Auckland New Zealand. Tyler loves mountain biking, particularly in upstate New York in Summer. Through his letters with Amelia, he has discovered that Amelia is a member of a mountain biking group that meets daily in Auckland.

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Tyler stopped reading the letter. He was puzzled. If the group climbed 5 kilometers, and it was 70° when they started, what was the temperature when they stopped?

First, let's think about the integers that we have here.

For every kilometer, there is a 2° **drop in temperature. So we can say that** $1\ km(-2) = -2^\circ$ **drop.**

The group traveled 5 kilometers, so we can take the 5 kilometers and multiply the distance by the number of degrees dropped per kilometer.

$5(-2) = -10^\circ$

It was 70° **when they started.**

$70 + -10 = 60^\circ$

It was 60° **when the group stopped their climb.**

You can learn more about Jafakids at www.aucklandmtb.co.nz/jafakids.

Vocabulary

Here are the vocabulary words that can be found in this lesson.

Product

the result of a multiplication problem.

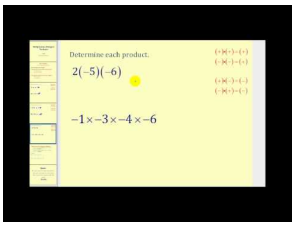
The Integers
the set of whole numbers and their opposites.

Commutative Property of Multiplication
a property that states that it doesn't matter which order you multiply terms. The product will be the same. $ab = ba$

Numerical Expression
an expression that contains multiple numbers and operations.

Algebraic Expression
an expression that contains numbers, variables and operations.

Technology Integration



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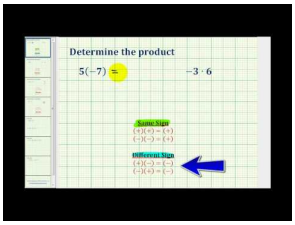
James Sousa,MultiplyingIntegers - The Basics



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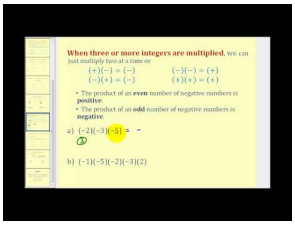
James Sousa,Exampleof Multiplying Integers



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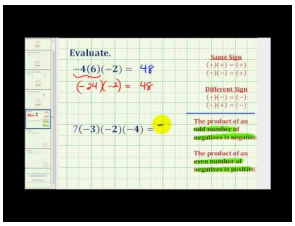
James Sousa,MultiplyingThree or More Integers



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James Sousa,Exampleof Multiplying Three orMore Integers

Other Videos:

1. http://teachertube.com/viewVideo.php?video_id=40423&title=integers_claymation – This is an engaging video using claymation about multiplying and dividing integers.

Time to Practice

Directions: Multiply each pair of integers to find a product.

1. (-7)(-8)
2. -3(4)
3. 5(8)
4. (-3)(-9)
5. 6(12)
6. -9(-9)
7. 8(-4)
8. -7(-2)
9. -7(-3)
10. 15(-2)
11. -15(2)
12. -2(-15)
13. 12(-5)
14. (-11)(-7)
15. (-4)(-5)
16. (-8)(-11)
17. (2)(-3)
18. -5(7)
19. -13(-2)
20. 14(2)

Directions: Evaluate each numerical expression.

21. (-9)(2)(-1)
22. (-3)(2)(-4)
23. (-5)(9)(-1)

24. (8)(-9)(-2)
25. (2)(-3)(-5)
26. -3(9 + 4)
27. -5(2 + 3 – 10)
28. -7(5 – 13)
29. -3(2) + (-4)(5)
30. (-8)(2) + (-3)(-2)