Inequalities

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CONCEPT 1 Inequalities

Name: __________________

a. $4 < 6$ (I think we can all agree on that, yes?)

    a. Add 4 to both sides of the equation. ___________ Is it still true?
    b. Add -4 to both sides of the (original) equation. ___________ Is it still true?
    c. Subtract 10 from both sides of the (original) equation. ___________ Is it still true?
    d. Multiply both sides of the (original) equation by 4. ___________ Is it still true?
    e. Divide both sides of the (original) equation by 2. ___________ Is it still true?
    f. Multiply both sides of the (original) equation by -3. ___________ Is it still true?
    g. *Divide* both sides of the (original) equation by -2. ___________ Is it still true?
    h. In general: what operations, when performed on an inequality, *reverse* the inequality?

b. $2x+3 < 7$

    a. Solve for x.
    b. Draw a number line below, and show where the solution set to this problem is.
    c. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $2x+3 < 7$. Does the inequality hold true?
    d. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $2x+3 < 7$. Does the inequality hold true?

c. $10-x \geq 4$

    a. Solve for x. Your first step should be adding x to both sides, so in your final equation, x is on the right side.
    b. Solve for x again from the original equation. This time, leave x on the left side.
    c. Did your two answers come out the same?
    d. Draw a number line below, and show where the solution set to this problem is.
    e. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $10-x \geq 4$. Does the inequality hold true?
    f. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $10-x \geq 4$. Does the inequality hold true?

d. $x = \pm 4$

    a. Rewrite this statement as two different statements, joined by "and" or "or."
    b. Draw a number line below, and show where the solution set to this problem is.

e. $-3 < x \leq 6$

    a. Rewrite this statement as two different statements, joined by "and" or "or."
    b. Draw a number line below, and show where the solution set to this problem is.

f. $x > 7 or\ x < -3$

    a. Draw a number line below, and show where the solution set to this problem is.

g. $x > 7 and\ x < -3$

    a. Draw a number line below, and show where the solution set to this problem is.

h. $x < 7 or\ x > -3$

    a. Draw a number line below, and show where the solution set to this problem is.

i. $x > \pm 4$

    a. Rewrite this statement as two different statements, joined by "and" or "or."

b. Draw a number line below, and show where the solution set to this problem is.

Name: __________________

Homework: Inequalities

1. $2x+7 \leq 4x+4$

a. Solve for x.

b. Draw a number line below, and show where the solution set to this problem is.

c. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $2x+7 \leq 4x+4$. Does the inequality hold true?

d. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $2x+7 \leq 4x+4$. Does the inequality hold true?

2. $14-2x < 20$

a. Solve for x.

b. Draw a number line below, and show where the solution set to this problem is.

c. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $14-2x < 20$. Does the inequality hold true?

d. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $14-2x < 20$. Does the inequality hold true?

3. $-10 < 3x+2 \leq 5$

a. Solve for x.

b. Draw a number line below, and show where the solution set to this problem is.

c. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $-10 < 3x+2 \leq 5$. Does the inequality hold true?

d. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $-10 < 3x+2 \leq 5$. Does the inequality hold true?

4. $x < 3 and\ x < 7$. Draw a number line below, and show where the solution set to this problem is.

5. $x < 3 or\ x < 7$. Draw a number line below, and show where the solution set to this problem is.

6. $x-2y \geq 4$

a. Solve for y.

b. Now—for the moment—let's pretend that your equation said *equals* instead of "greater than" or "less than." Then it would be the equation for a line. Find the slope and the $y-$intercept of that line, and graph it.

Slope: __________

$y-$Intercept: __________

c. Now, pick any point (x,y) that is *above* that line. Plug the x and y coordinates into your inequality from part (a). Does this point fit the inequality? (Show your work...)

d. Now, pick any point (x,y) that is *below* that line. Plug the x and y coordinates into your inequality from part (a). Does this point fit the inequality? (Show your work...)

e. So, is the solution to the inequality the points *below* or *above* the line? Shade the appropriate region on your graph.

7. Using a similar technique, draw the graph of $y \geq x^2$. (If you don't remember what the graph of $y = x^2$ looks like,

try plotting a few points!)

Name: ____________________

Inequality Word Problems

a. Jacob is giving a party. 20 people showed up, but he only ordered 4 pizzas! Fortunately, Jacob hasn't *cut* the pizzas yet. He is going to cut each pizza into n slices, and he needs to make sure there are enough slices for everyone at the party to get at least one. Write an inequality or set that describes what n has to be.
b. Whitney wants to drive to Seattle. She needs 100 gallons of gas to make the trip, but she has only $80 allocated for gas. Her strategy is to wait until the price of gas is low enough that she can make the trip. Write an inequality or set that describes what the price of gas *has to be* for Whitney to be able to reach Seattle. Be sure to clearly define your variable(s)!
c. Your evil math teacher, who shall go nameless, is only giving two tests for the whole grading period. They count equally—your grade will be the average of the two. Your first test was a 90. Write an inequality or set that describes what your *second* test grade has to be, in order for you to bring home an A on your report card. ("A" means 93 or above.) Be sure to clearly define your variable(s)!
d. Laura L is going to build a movie theater with n screens. At each screen, there will be 200 seats for the audience to watch that movie. (So maximum capacity is 200 audience members per screen.) In addition to audience members, there are 20 employees on the premises at any given time (selling tickets and popcorn and so on). According to code (which I am making up), she must have at least one bathroom for each 100 people in the building. (Of course, it's fine to build more bathrooms than that, if she wants!)
    a. Write a function (this will be an equation) relating the number of screens (n) to the total number of people who can possibly be in the building (p). Which one is dependent? Which one is independent?
    b. Write an *inequality* relating the total number of people who can possibly be in the building (p) to the number of bathrooms (b).
    c. Now write a *composite inequality* (I just made that word up) that tells Laura: if you build this many screens, here is how many bathrooms you need.
e. Make up your own word problem for which the solution is an inequality, and solve it. The topic should be *breakfast*.

Name:____________________

Absolute Value Equations

1. $|4| =$

2. $|-5| =$

3. $|0| =$

4. OK, now, I'm thinking of a number. All I will tell you is that the *absolute value of my number is* 7.

a. Rewrite my question as a math equation instead of a word problem.

b. What can my number be?

5. I'm thinking of a different number. This time, the *absolute value of my number is* 0.

a. Rewrite my question as a math equation instead of a word problem.

b. What can my number be?

6. I'm thinking of a different number. This time, the *absolute value of my number is* -4.

a. Rewrite my question as a math equation instead of a word problem.

b. What can my number be?

7. I'm thinking of a different number. This time, the *absolute value of my number is less than* 7.

a. Rewrite my question as a math inequality instead of a word problem.

b. Does 8 work?

c. Does 6 work?

d. Does -8 work?

e. Does -6 work?

f. Write an inequality that describes all possible values for my number.

8. I'm thinking of a different number. This time, the *absolute value of my number is greater than* 4.

a. Rewrite my question as a math inequality instead of a word problem.

b. Write an inequality that describes all possible values for my number. (Try a few numbers, as we did in 7.)

9. I'm thinking of a different number. This time, the *absolute value of my number is greater than* -4.

a. Rewrite my question as a math inequality instead of a word problem.

b. Write an inequality that describes all possible values for my number.

Stop at this point and check your answers with me before going on to the next side.

10. $|x+3|=7$

a. First, forget that it says, [U+0080][U+009C]$x+3$[U+0080][U+009D] and just think of it as "a number." The absolute value of *this number* is 7. So what can *this number* be?

b. Now, remember that "this number" is $x+3$. So write an equation that says that $x+3$ can be <*your answer(s) in part a*>.

c. Solve the equation(s) to find what x can be.

d. Plug your answer(s) back into the original equation $|x+3|=7$ and see if they work.

11. $4|3x-2|+5=17$

a. This time, because the absolute value is not alone, we're going to start with some algebra. Leave $|3x-2|$ alone, but get rid of everything *around* it, so you end up with $|3x-2|$ alone on the left side, and some other number on the right.

b. Now, remember that "some number" is $3x-2$. So write an equation that says that $3x-2$ can be <*your answer(s) in part a*>.

c. Solve the equation(s) to find what x can be.

d. Plug your answer(s) back into the original equation $4|3x-2|+5=17$ and see if they work.

12. $|x-3|+5=4$

a. Solve, by analogy to the way you solved the last two problems.

b. Plug your answer(s) back into the original equation $|x-3|+5=4$ and see if they work.

13. $|x-2|=2x-10$.

a. Solve, by analogy to the way you solved the last two problems.

b. Plug your answer(s) back into the original equation $|x-2|=2x-10$ and see if they work.

Name: ___________________

Homework: Absolute Value Equations

a. $|x| = 5$
   a. Solve for x
   b. Check your answer(s) in the original equation.
b. $|x| = 0$
   a. Solve for x
   b. Check your answer(s) in the original equation.
c. $|x| = -2$
   a. Solve for x
   b. Check your answer(s) in the original equation.
d. $10|x| = 5$
   a. Solve for x
   b. Check your answer(s) in the original equation.
e. $|x+3| = 1$
   a. Solve for x
   b. Check your answer(s) in the original equation.
f. $\frac{4|x-2|}{3} = 2$
   a. Solve for x
   b. Check your answer(s) in the original equation.
g. $7|2x+3|-4 = 4$
   a. Solve for x
   b. Check your answer(s) in the original equation.
h. $|2x-3| = x$
   a. Solve for x
   b. Check your answer(s) in the original equation.
i. $|2x+2| = x$
   a. Solve for x
   b. Check your answer(s) in the original equation.
j. $|x-5| = 2x-7$
   a. Solve for x
   b. Check your answer(s) in the original equation.

Check-yourself hint: for numbers 8, 9, *and* 10, *one of them has* <u>*no*</u> *valid solutions,* <u>*one*</u> *has one valid solution, and one has* <u>*two*</u> *valid solutions*.

Name: ___________________

Absolute Value Inequalities

a. $|x| \leq 7$
   a. Solve.
   b. Graph your solution on a number line
   c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

b. $|2x+3| \leq 7$

   a. Write down the solution for what $2x+3$ has to be. This should look exactly like your answer to number 1, except with a $(2x+3)$ instead of an (x).
   b. Now, solve that inequality for x.
   c. Graph your solution on a number line
   d. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   e. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

c. $4|3x-6|+7 > 19$

   a. Solve for $|3x-6|$. (That is, leave the $|3x-6|$ part alone, but get rid of all the stuff around it.)
   b. Write down the inequality for what $(3x-6)$ has to be.
   c. Now, solve that inequality for x.
   d. Graph your solution on a number line
   e. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   f. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

d. $\frac{|3x-4|}{2}+6 < 3$

   a. Solve for x. (You know the drill by now!)
   b. Graph your solution on a number line
   c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

e. $6|2x^2-17x-85|+5 \geq 3$

   a. Solve for x.
   b. Graph your solution on a number line
   c. hoose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

Name: ___________________

Homework—Absolute Value Inequalities

a. $|4+3x| = 2+5x$ (...OK, this isn't an *in*equality, but I figured you could use a bit more practice at these)
b. $|x| = x-1$
c. $4|2x-3|-5 \geq 3$

   a. Solve for x.
   b. Graph your solution on a number line
   c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

d. $3|x-5|+2 < 17$

   a. Solve for x.
   b. Graph your solution on a number line
   c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

e. $-3|x-5|+2 < 17$

   a. Solve for x.
   b. Graph your solution on a number line
   c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
   d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

f. $2|x+2|+6 < 6$

a. Solve for x.
b. Graph your solution on a number line
c. Choose a point that *is* in your solution set, and test it in the original inequality. Does it work?
d. Choose a point that is *not* in your solution set, and test it in the original inequality. Does it work?

Name: __________________

Graphing Inequalities and Absolute Values

a. $9x+3y \leq 6$
   a. Put into a sort of $y = mx+b$ format, except that it will be an *in*equality.
   b. Now, ignore the fact that it is an inequality—pretend it is a line, and graph that line.
   c. Now, to graph the inequality, shade in the area either *above* the line, or *below* the line, as appropriate. (Hint: does y have to be *less than* the values on the line, or *greater than* them?)
   d. Test your answer. Choose a point (any point) in the region you shaded, and test it in the inequality. Does the inequality work? (Show your work.)
   e. Choose a point (any point) in the region you did *not* shade, and test it in the inequality. Does the inequality work? (Show your work.)
b. $4x \leq 2y+5$
   a. Graph the inequality, using the same steps as above.
   b. Test your answer by choosing one point *in* the shaded region, and one point that is *not* in the shaded region. Do they give you the answers they should? (Show your work.)
c. $y = |x|$
   a. Create a table of points. Your table should include at least two positive $x-$values, two negative $x-$values, and $x = 0$.
   b. Graph those points, and then draw the function.
d. $y = |x|+3$. Graph this *without* a table of points, by remembering what "adding 3" does to any graph. (In other words, what will these $y-$values be like compared to your $y-$values in 3?)
e. $y = -|x|$. Graph this *without* a table of points, by remembering what "multiplying by -1" does to any graph. (In other words, what will these $y-$values be like compared to your $y-$values in 3?)
f. *Now, let's put it all together!!!*
   a. Graph it. $y = -|x|+2$
   b. Graph $y \geq -|x|+2$. Your answer will *either* be a shaded region on a $2-$dimensional graph, *or* on a number line.
   c. Test your answer by choosing one point *in* the shaded region, and one point that is *not* in the shaded region. Do they give you the answers they should? (Show your work.)
   d. Graph $-|x|+2 < 0$. Your answer will *either* be a shaded region on a $2-$dimensional graph, *or* on a number line.
   e. Test your answer by choosing one point *in* the shaded region, and one point that is *not* in the shaded region. Do they give you the answers they should? (Show your work.)
g. *Extra for experts:* $y \geq 3|x+4|$
   a. Graph it. Think hard about what that $+4$ and that 3 will do. Generate a few points if it will help you!
   b. Test your answer by choosing one point *in* the shaded region, and one point that is *not* in the shaded region. Do they give you the answers they should? (Show your work.)

Name: __________________

Homework: Graphing Inequalities and Absolute Values

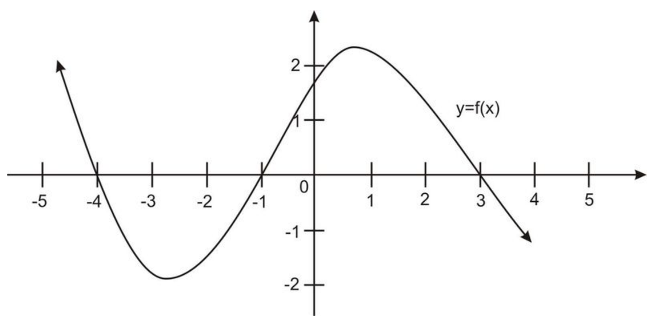
1. The famous detectives Guy Noir and Nick Danger are having a contest to see who is better at catching bad guys. At 8:00 in the evening, they start prowling the streets of the city. They have twelve hours. Each of them gets 10 points for every purse-snatcher he catches, and 15 points for every cat-burglar. At 8:00 the next morning, they meet in one of their dingy offices to compare notes. "I got 100 points," brags Nick. If Guy gets enough snatchers and burglars, he will win the contest.

a. Label and clearly describe the relevant variables.

b. Write an inequality relating the variables you listed in part (a). I should be able to read it as, "If this inequality is true, then Guy wins the contest."

c. Graph the inequality from part (b).

2. The graph below shows the function $y = f(x)$.



a. Graph $y \leq f(x)$. Your answer will *either* be a shaded region on a 2−dimensional graph, *or* on a number line.

b. Graph $f(x) < 0$. Your answer will *either* be a shaded region on a 2−dimensional graph, *or* on a number line.

3. $x - 2y > 4$

a. Graph.

b. Pick a point in your shaded region, and plug it back into our original equation $x - 2y > 4$. Does the inequality work? (Show your work!)

c. Pick a point which is *not* in your shaded region, and plug it into our original equation $x - 2y > 4$. Does the inequality work? (Show your work!)

4. $|x| - y \geq 2$

a. Graph.

b. Pick a point in your shaded region, and plug it back into our original equation $|x| - y \geq 2$. Does the inequality work? (Show your work!)

c. Pick a point which is *not* in your shaded region, and plug it into our original equation $|x| - y \geq 2$. Does the inequality work? (Show your work!)

5. $y > x^3$

a. Graph. (Plot points to get the shape.)

b. Pick a point in your shaded region, and plug it back into our original equation $y > x^3$. Does the inequality work? (Show your work!)

c. Pick a point which is *not* in your shaded region, and plug it into our original equation $y > x^3$. Does the inequality work? (Show your work!)

6. Graph: $y + |x| < -|x|$. Think hard—you can do it!

Name: ___________________

Sample Test: Inequalities and Absolute Values

a. $1 < 4 - 3x \leq 10$
   a. Solve for x.
   b. Draw a number line below, and show where the solution set to this problem is.
   c. Pick an $x-$value which, according to your drawing, is *inside* the solution set. Plug it into the original inequality $1 < 4 - 3x \leq 10$. Does the inequality hold true? (Show your work!)
   d. Pick an $x-$value which, according to your drawing, is *outside* the solution set. Plug it into the original inequality $1 < 4 - 3x \leq 10$. Does the inequality hold true? (Show your work!)
b. Find the x value(s) that make this equation true: $4|2x+5|-3 = 17$
c. Find the x value(s) that make this equation true: $|5x - 23| = 21 - 6x$
d. $\frac{|2x-3|}{3} + 7 > 9$
   a. Solve for x.
   b. Show graphically where the solution set to this problem is.
e. $-3|x+4|+7 \geq 7$
   a. Solve for x.
   b. Show graphically where the solution set to this problem is.
f. Make up and solve an inequality word problem having to do with *hair*.
   a. Describe the scenario in words.
   b. Label and clearly describe the variable or variables.
   c. Write the inequality. (Your answer here should be *completely determined* by your answers to (a) and (b) - I should know exactly what you're going to write. If it is not, you probably did not give enough information in your scenario.)
g. Graph $y \geq -|x|+2$
h. $|2y|-|x| > 6$
   a. Rewrite this as an inequality with *no absolute values,* for the *fourth quadrant* (lower-right-hand corner of the graph).
   b. Graph what this looks like, in the fourth quadrant only.
i. Graph $y = x - |x|$