Dividing Integers

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CONCEPT 1 Dividing Integers

Introduction

The History Test

Jessica couldn't believe that her pen pal Harrison had a history test the exact same day that she did. Wow! Imagine the coincidence of having a test in the same subject on the same day. Jessica went to class excited to find out about her score on the test. She wondered how her score would compare to Harrison's test score.

Mr. Watson, the history teacher, asked each student to come up to his desk to see his or her score. He wanted to hold on to the test papers, but wanted the students to be able to see how they had done on the test.

Jessica walked up to Mr. Watson's desk and looked at the paper. She saw that she had earned a 90% on the 100 point test.

Jessica was very excited.

"How much was each question worth?" She asked Mr. Watson.

"Each question was worth 2 points, out of the possible 100," Mr. Watson told Jessica.

Jessica went back to her seat and couldn't help smiling. Then she started to think about the score. If she had earned a 90% on the test and each question had been worth 2 points, then how many questions had she answered incorrectly?

If Jessica earned a 90%, then that is -10 out of 100.

If each question missed took 2 points off of the test score, how many questions did Jessica answer incorrectly?

To help Jessica with this dilemma, you will need to understand how to divide integers. In this lesson you will learn all that you need to know about dividing integers. Take what you have learned to help Jessica at the end of this lesson.

What You Will Learn

By the end of this lesson, you will be able to demonstrate the following skills:

- Find quotients of positive integers.
- Find quotients of positive and negative integers.
- Find quotients of negative integers.
- Evaluate numerical and algebraic expressions involving integer division.

Teaching Time

I. **Find Quotients of Positive Integers**

Having just finished learning about the multiplication of integers, it makes the most sense to move on to learning about the division of integers. Let's start by learning about dividing positive integers.

What is a quotient?

A **quotient** is the answer in a division problem.

You have been dividing numbers for a long time. Remember that dividing means that we are splitting up a quantity into groups. Here is a picture of 10 divided by two.

There are ten suns pictured here. We have divided the ten suns into two rows. There are five in each row. The grouping is the row. The answer is five.

When we divide ten into two groups, we have five in each group.

$10 \div 2 = 5$

Five is the quotient of the problem. It is the answer.

We can divide positive integers. Dividing positive integers is the same as dividing a positive whole number.

Let's practice a few examples.

Example

$15 \div 3 =$ ____

Fifteen divided by three is five.

The answer is five.

Example

$\frac{25}{5}$

Ah, that is a good question. This looks like a fraction, but in this example, the fraction bar is being used to show division. We can show division using the regular division symbol, ÷, or we can show division using a fraction bar.

This problem means twenty-five divided by five.

$25 \div 5 = 5$

The answer is five.

From these examples, we can write the following rule about dividing positive integers.

Positive ÷ Positive = Positive

Practice a few of these on your own. Find each quotient.

1. $81 \div 9 =$ ____
2. $\frac{64}{2}$
3. $\frac{39}{3}$

Take a few minutes to check your work with a friend. Do your answers match?

II. **Find Quotients of Positive and Negative Integers**

To understand quotients of positive and negative integers, it may help to think of division as the opposite of multiplication. **Remember that multiplication and division are inverse operations. Inverse operation** means opposite operation.

If we multiply a negative number and a positive number, we already learned that we get a negative answer.

Example

$-3(9) = -27$

We can think of this problem as a division problem using the inverse operation of multiplication, division.

Example

$-27 \div 9 = -3$

A negative divided by a positive is equal to a negative number.

Using this example, we can write a rule about dividing negative and positive integers.

A negative divided by a positive is a negative.

A positive divided by a negative is a negative.

Whenever integers of different signs are divided, the quotient is negative.

Let's look at an example.

Example

-36 ÷ 4 = ____

Negative thirty-six divided by four is -9. A negative divided by a positive is a negative answer.

The quotient of this problem is -9.

As long as you remember the rule, it is quite simple to divide negative integers by positive integers and positive integers by negative integers.

Practice a few of these on your own.

1. **-18 ÷ 6 = ____**
2. **26 ÷ -13 = ____**
3. **-49 ÷ 7 = ____**

Check your work. Is your work accurate?

III. **Find Quotients of Negative Integers**

To find quotients of negative integers we can think about dividing up quantities into groups. Let's look at this example.

Example

Divide -12 into groups of -4. How many groups do you have?

To do this, we have to think of 12 units worth -1 each.

We can say that each red circle has a value of -1. Therefore, the value of this entire group is -12. Now we want to divide this into groups of -4. That means each group will have four red circles in them. Let's reorganize the red circles and see how many groups of -4 we have.

We end up with three groups of negative four.

-12 ÷ -4 = 3

A negative number divided by another negative number is a positive. If you think about this, we can't divide a negative number into groups of negatives and have negative groups. It doesn't make sense. We divided -12 into groups of -4 and ended up with 3 groups. The number of groups is positive.

Here is our rule.

Negative ÷ negative = positive

Use this rule to practice finding quotients.

1. **-16 ÷ -2 = ____**
2. **-24 ÷ -12 =____**
3. **-64 ÷ -2 = ____**

Take a few minutes to check your work.

Write down the rules for dividing integers into your notebook.

IV. **Evaluate Numerical and Algebraic Expressions Involving Integer Division**

Before we begin evaluating expressions involving integer division, let's take a minute to review the rules for dividing integers.

One of the best ways to become great at integer multiplication and division is to memorize the rules for multiplying and dividing. Using your memory will aid you in solving problems from now on when you tackle integers in mathematics.

Take a few minutes to see if you can remember the rules for multiplying and dividing integers without looking. Quiz a partner and see if you can help each other remember these very important rules!!

Now let's move on to evaluating expressions involving integer division. Remember you will need to apply the rules for dividing integers in many different situations.

We can start with evaluating numerical expressions.

What is a numerical expression?

A **numerical expression** is an expression that contains several integers and at least two operations. We can figure out the value of the expression by using the information that we have learned. Here is an example.

Example

$\frac{-8(-9)}{8}$

Here we have multiplication on the top of the fraction bar and the number eight on the bottom. **Remember that the fraction bar means division!**

To evaluate this expression, we complete the multiplication first, then the division.

$-8 \times -9 = 72$**8 and a negative** *x***a negative is a positive**

$$\frac{72}{8}$$
$$72 \div 8 = 9$$

The answer is 9.

Sometimes we can have addition or subtraction with division too.

Example

$\frac{6+-10}{2}$

First, we complete the addition on the top of the fraction bar.

6 + -10 = -4

Now we complete the division.

$$\frac{-4}{2} = -2$$

Negative four divided by two is negative two.

The answer is -2.

How do we evaluate an algebraic expression?

Remember that an **algebraic expression** combines variables, integers and operations. While learning, you will often be given a value for the variable. Then you substitute the given value into the expression and evaluate it.

Example

$xy \div (-4)$ *when* $x = 2$ *and* $y = 8$

Here we have xy next to each other which means multiplication. We can substitute the given values in for x and y. We do this first.

(2)(8) ÷ (-4)

Next, we follow the order of operations and complete multiplication/division in order from left to right.

$$16 \div (-4)$$
$$-4$$

The answer is -4.

Always remember to follow the integer rules you have learned when evaluating expressions.

Practice evaluating the following expressions.

1. $\frac{-9+3}{2}$
2. $\frac{4x}{2}$ *when* $x = -6$
3. $\frac{-14(3)}{7}$

Take a few minutes to check your work with a partner.

Real Life Example Completed

The History Test

Here is the original problem once again. Take a few minutes to reread the problem and underline any important information.

Jessica couldn't believe that her pen pal Harrison had a history test the exact same day that she did. Wow! Imagine the coincidence of having a test in the same subject on the same day. Jessica went to class excited to find out about her score on the test. She wondered how her score would compare to Harrison's test score.

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If Jessica earned a 90%, then that is -10 out of 100.

If each question missed took 2 points off of the test score, how many questions did Jessica answer incorrectly?

Now let's think about this question. We can begin by writing down the integers that we will be working with.

Jessica had 10 points taken off of her score = -10

Each question was worth 2 points, so 2 points were taken off by each wrong question = -2

Next, we divide the total number of points off by the number of questions that were incorrectly answered.

-10 ÷ -2 = 5

A negative divided by a negative is a positive.

Jessica answered 5 questions incorrectly.

Vocabulary

Here are the vocabulary words that are used in this lesson.

Quotient
the answer from a division problem

Integer
the set of whole numbers and their opposites

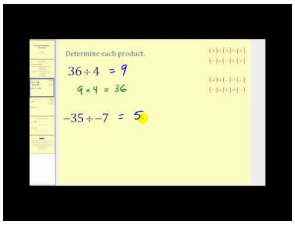
Inverse Operation
the opposite of a given operation

Fraction Bar
the line used to divide the numerator and the denominator of a fraction-also means division

Numerical expression
an expression that combines integers and operations

Algebraic expression
an expression that combines variables, integers and operations.

Technology Integration



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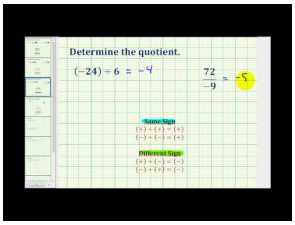
James Sousa,Division ofIntegers - The Basics



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James Sousa,Exampleof DividingIntegers

Other Videos:

1. http://teachertube.com/viewVideo.php?video_id=40423&title=integers_claymation – This is a teachertube video that uses claymation to explain the rules for dividing and multiplying integers.

Time to Practice

Directions: Find the quotient of each integer pair.

1. -18 ÷ 9 = ____
2. -22 ÷ -11 = ____
3. -32 ÷ 8 = ____
4. 32 ÷ 8 = ____
5. -21 ÷ 7 = ____
6. -72 ÷ 12 = ____
7. -80 ÷ -10 = ____
8. 56 ÷ -7 = ____
9. 63 ÷ -9 = ____
10. -121 ÷ -11 = ____
11. 144 ÷ -12 = ____
12. 200 ÷ -4 = ____
13. -50 ÷ -2 = ____
14. 28 ÷ -2 = ____
15. 66 ÷ 3 = ____
16. 150 ÷ -3 = ____
17. 180 ÷ -90 = ____
18. 70 ÷ -35 = ____
19. -44 ÷ -22 = ____
20. 75 ÷ 3 = ____

Directions: Evaluate each numerical expression.

21. $\frac{-9+-3}{6}$
22. $\frac{-9(-6)}{2}$
23. $\frac{(15)(3)}{-5}$
24. $\frac{-18(4)}{9}$
25. $\frac{-3-12}{-5}$