PROBLEM ANSWERS Chapter Five

Section 5.1

1. (a) $(10)(12) + (8)(4) = 152$ (b) $(10)(20) - (3)(8) = 176$

3. $bh + \frac{1}{2}\, b(H - h) = bh + \frac{1}{2}\, bH - \frac{1}{2}\, bh = b\left(\frac{h + H}{2}\right)$

5. (a) $(1)(3) + (1)(2) = 5$ (b) {area of shaded region in Fig. 24b} < 5

7. $A(1) = 1,\ A(2) = 2.5,\ A(3) = 4.5,\ A(4) = 6,\ A(5) = 7$

9. $C(1) = 1.5,\ C(2) = 4,\ C(3) = 7.5$, and $C(x) =$ rect. + triangle areas $= x + \frac{1}{2}\, x \cdot x = x + \frac{1}{2}\, x^2$

11. Distance = "area" $= (20)(30) + \frac{1}{2}(10)(30) = 600 + 150 = 750$ feet.

13. (a) A: 20 seconds to stop. B: 40 seconds to stop.
(b) A: $\frac{1}{2}(20)(80) = 800$ feet to stop. B: $\frac{1}{2}(40)(40) = 800$ feet to stop.

15. miles, dollars, cubic feet, kilowatt•hours, people, square meals

Section 5.2

1. $2^2 + 3^2 + 4^2 = 29$ 3. $(1+1)^2 + (1+2)^2 + (1+3)^2 = 29$

5. $\cos(0) + \cos(\pi) + \cos(2\pi) + \cos(3\pi) + \cos(4\pi) + \cos(5\pi) = 1 + (-1) + 1 + (-1) + 1 + (-1) = 0$

7. $\sum_{k=3}^{94} k$ 9. $\sum_{k=3}^{12} k^2$ 11. $\sum_{k=1}^{7} k \cdot 2^k$

13. (a) $(1+2) + (2+2) + (3+2) = 3 + 4 + 5 = 12$ (b) $(1+2+3) + (2+2+2) = 12$

15. (a) $5 \cdot 1 + 5 \cdot 2 + 5 \cdot 3 = 5 + 10 + 15 = 30$ (b) $5 \cdot (1 + 2 + 3) = 5 \cdot 6 = 30$

17. (a) $1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$ (b) $(1 + 2 + 3)^2 = 6^2 = 36$

19. $f(0) + f(1) + f(2) + f(3) = 0^2 + 1^2 + 2^2 + 3^2 = 14$

21. $2 \cdot f(0) + 2 \cdot f(1) + 2 \cdot f(2) + 2 \cdot f(3) = 2 \cdot 0 + 2 \cdot 1 + 2 \cdot 4 + 2 \cdot 9 = 28$

23. $g(1) + g(2) + g(3) = 3 + 6 + 9 = 18$

25. $g^2(1) + g^2(2) + g^2(3) = 3^2 + 6^2 + 9^2 = 126$

27. $h(2) + h(3) + h(4) = \frac{2}{2} + \frac{2}{3} + \frac{2}{4} = \frac{13}{6}$

Source URL: http://scidiv.bellevuecollege.edu/dh/Calculus_all/Calculus_all.html
Saylor URL: http://www.saylor.org/courses/ma005/

29 $f(1)h(1) + f(2)h(2) + f(3)h(3) = (1)(2) + (4)(1) + (9)(2/3) = 12$

31. $(1^2 - 0^2) + (2^2 - 1^2) + (3^2 - 2^2) + (4^2 - 3^2) + \ldots + (7^2 - 6^2) = 7^2 - 0^2 = 49$

33. $(\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{6}) = 1 - \frac{1}{6} = \frac{5}{6}$

35. $(\sqrt{1} - \sqrt{0}) + (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + \ldots + (\sqrt{9} - \sqrt{8}) = \sqrt{9} - \sqrt{0} = 3$

37. (i) $[2,3], [3,4.5], [4.5,6], [6,7]$ (ii) $1, 1.5, 1.5, 1$
(iii) mesh = 1.5 (iv) $1 + 1.5 + 1.5 + 1 = 5$

39. (i) $[-3,-1], [-1,0], [0,1.5], [1.5,2]$ (ii) $2, 1, 1.5, 0.5$
(iii) mesh = 2 (iv) $2 + 1 + 1.5 + 0.5 = 5$

41. (i) $[3, 3.8], [3.8, 4.5], [4.5, 5.2], [5.2, 7]$ (ii) $0.8, 0.7, 0.7, 1.8$ (iii) mesh = 1.8 (iv) $0.8+0.7+0.7+1.8 = 4$

43. $\Delta x_1 + \Delta x_2 + \Delta x_3 + \ldots + \Delta x_n = (x_1 - x_0) + (x_2 - x_1) + (x_3 - x_2) + \ldots + (x_n - x_{n-1}) = x_n - x_0$

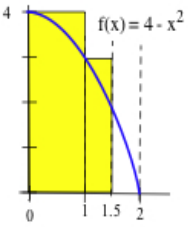


Fig. for 45(a)

45. (a) $f(0)(1) + f(1)(0.5) + f(2)(0.5) = (4)(1) + (3)(0.5) + (0)(0.5) = 5.5$
(b) $f(1)(1) + f(1.5)(0.5) + f(1.5)(0.5) = (3)(1) + (1.75)(0.5) + (1.75)(0.5) = 4.75$

47a. (i) and (ii) See the graph.
(iii) $f(0) = 0$, $f(\pi/4) \approx 0.707$, $f(\pi/2) = 1$
(iv) $(\pi/4)(0) + (\pi/4)(0.707) + (\pi/2)(1) \approx 2.13$

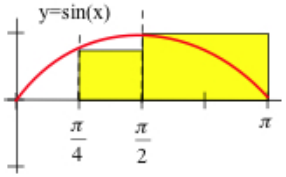


Fig. for 47(a)

49. (a) $(2)(1) + (5)(2) + (17)(1) \le RS \le (5)(1) + (17)(2) + (26)(1)$ so $29 \le RS \le 65$.
(b) $(2)(1) + (5)(1) + (10)(1) + (17)(1) \le RS \le (5)(1) + (10)(1) + (17)(1) + (26)(1)$ so $34 \le RS \le 58$.
(c) $(2)(0.5) + (3.25)(0.5) + (5)(1) + (10)(1) + (17)(1) \le RS \le (3.25)(0.5) + (5)(0.5) + (10)(1) + (17)(1) + (26)(1)$ so $34.625 \le RS \le 57.125$.

51. (a) $(0)(\pi/2) + (0)(\pi/2) \le RS \le (1)(\pi/2) + (1)(\pi/2)$ so $0 \le RS \le \pi$
(b) $(0)(\pi/4) + (0.707)(\pi/4) + (0)(\pi/2) \le RS \le (0.707)(\pi/4) + (1)(\pi/4) + (1)(\pi/2)$ so $0.56 \le RS \le 2.91$
(c) $(0)(\pi/4) + (0.707)(\pi/\ ) + (0.707)(\pi/4) + (0)(\pi/4) \le RS \le (0.707)(\pi/4) + (1)(\pi/4) + (1)(\pi/4) + (0.707)(\pi/4)$ so $1.11 \le RS \le 2.68$

53. (a) $|7.402 - 7.362| = 0.04$ (b) $|7.390 - 7.372| = 0.018$

55. $|\text{error}| \le (\text{base})(\text{height}) = \frac{4-2}{50}(65-9) = \frac{2}{50}(56) = \frac{112}{50} = 2.24$

57. (a) $\frac{100(101)}{2} = 5050$ (b) $1 + 2 + 3 + \ldots + 100$
$\underline{100 + 99 + 98 + \ldots + 1}$
$101 + 101 + 101 + \ldots + 101 = 100(101) = 10100$. $\frac{1}{2}(10100) = 5050$

59. $10 + 11 + 12 + \ldots + 20 = (1+2+3 + \ldots + 20) - (1+2+3 + \ldots + 9) = \frac{20(21)}{2} - \frac{9(10)}{2} = 210 - 45 = 165$

61. $\sum_{k=1}^{10}(k^3+k) = \sum_{k=1}^{10}k^3 + \sum_{k=1}^{10}k = \left(\frac{10(11)}{2}\right)^2 + \frac{10(11)}{2} = (55)^2 + 55 = 3080$

Section 5.3

1. $\int_0^4 2+3x\ dx$

3. $\int_2^5 x^3\ dx$

5. $\int_1^5 x^3\ dx$

7. $\int_{0.5}^2 x\sin(x)\ dx$

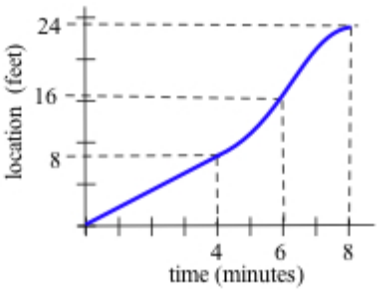
9. $\int_1^3 \ln(x)\ dx$

11. $\int_1^3 2x\ dx = 8$

13. $\int_{-1}^0 |x|\ dx = 1/2$

15. $\int_0^4 3-\frac{x}{2}\ dx = 8$

Fig. for 19(a)



17. (a) 3 (b) –1 (c) 6 (d) 8 (e) 7

19. (a) see the graph
(b) 24 feet.
(c) 24 feet from the starting point.

21. meters

23. feet^3 = cubic feet

25. gram•meters

27. feet/second = feet per second

29. $\Delta x = \frac{2-0}{n} = \frac{2}{n}$. $m_i = \frac{2}{n}(i-1)$ and $M_i = \frac{2}{n}i$ so $f(m_i) = \{\frac{2}{n}(i-1)\}^3$ and $f(M_i) = \{\frac{2}{n}i\}^3$.

(a) $LS = \sum_{i=1}^n f(m_i)\Delta x = \sum_{i=1}^n \{\frac{2}{n}(i-1)\}^3\frac{2}{n} = \frac{2}{n}\frac{8}{n^3}\left\{\sum_{i=1}^n i^3 - 3\sum_{i=1}^n i^2 + 3\sum_{i=1}^n i - \sum_{i=1}^n 1\right\}$

$= \frac{16}{n^4}\left\{\left(\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{3}{12}n^2\right) - 3\left(\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{2}{12}n\right) + 3\left(\frac{1}{2}n^2+\frac{1}{2}n\right) - n\right\}$

$= \frac{16}{n^4}\left\{\frac{1}{4}n^4 - \frac{1}{2}n^3 + \frac{1}{4}n^2\right\} = 4 - \frac{8}{n} + \frac{4}{n^2} \longrightarrow 4$.

(b) $US = \sum_{i=1}^n f(M_i)\Delta x = \sum_{i=1}^n \{\frac{2}{n}i\}^3\frac{2}{n} = \frac{2}{n}\frac{8}{n^3}\left\{\sum_{i=1}^n i^3\right\} = \frac{16}{n^4}\left\{\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{3}{12}n^2\right\}$

$= 4 + \frac{8}{n} + \frac{4}{n^2} \longrightarrow 4$.

Section 5.4

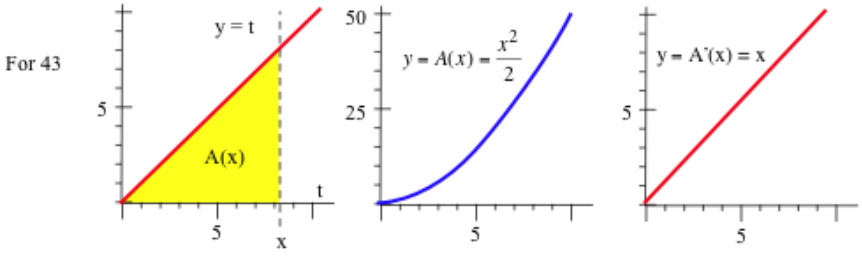
1. 5
3. 0
5. 3
7. 0
9. –5
11. –5
13. 0
15. 4.5+5 = 9.5
17. 10+3 = 13
19. 5+2 = 7
21. 1
23. –1

25. 2.5 27. 1 29. 1 31. (a) $8 \cdot 6 = 48$ (b) 24

33. (a) 32 (b) $8^2 = 64$ 35. 8 37. 2.5

39. 3 41. 7

43. (a) graph $y = A(x) = x^2/2$ (b) graph $y = A'(x) = x$

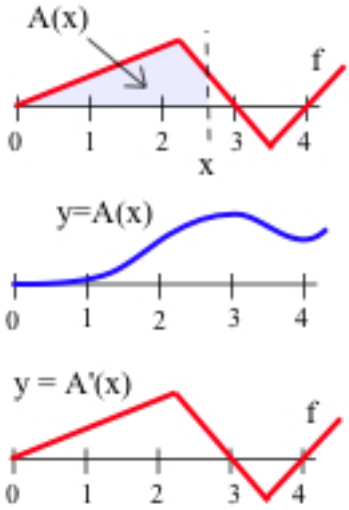


45. (a) graph of $y = A(x)$ in Figure 45a
(b) graph of $y = A'(x)$ in Figure 45b

47. (a) f is continuous on [1,4]
(b) f is not differentiable on [1,4] (not diff. at $x \approx 2.3$ and 3.3)
(c) f is integrable on [1,4]

49. (a) f is not continuous on [1,4] (not cont. at x = 2 and 3)
(b) f is not differentiable on [1,4] (not diff. at x = 2 and 3)
(c) f is integrable on [1,4].

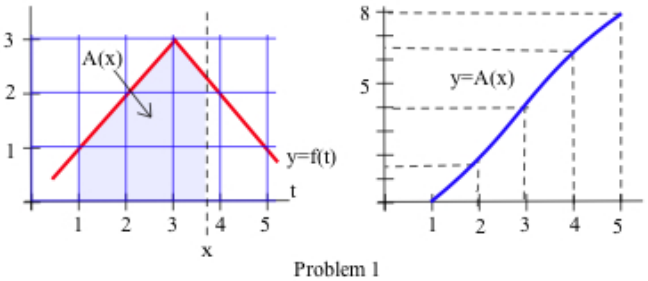
51. $\int_1^4$ velocity dt = $\int_1^2$ velocity dt + $\int_2^4$ velocity dt = 35 + 50 = 85 miles



Graphs for problem 45

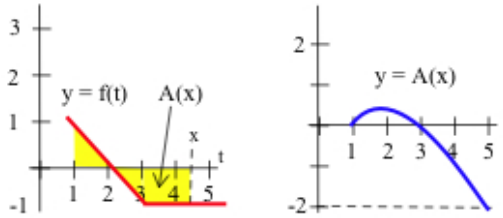
Section 5.5

1. (a) see Figure
(b) A(1) = 0 , A(2) = 1.5 , A(3) = 4 , A(4) = 6.5
(c) A '(1) = 1 , A '(2) = 2 , A '(3) = 3 , A '(4) = 2



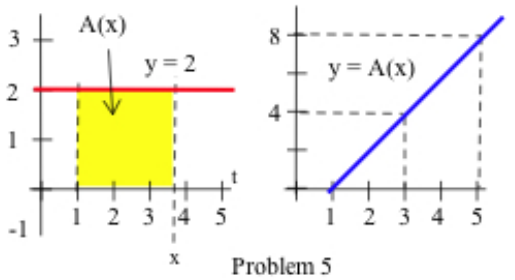
Problem 1

3. (a) see Figure
(b) A(1) = 0 , A(2) = 0.5 , A(3) = 0 , A(4) = –1
(c) A '(1) = 1 , A '(2) = 0 , A '(3) = –1 , A '(4) = –1



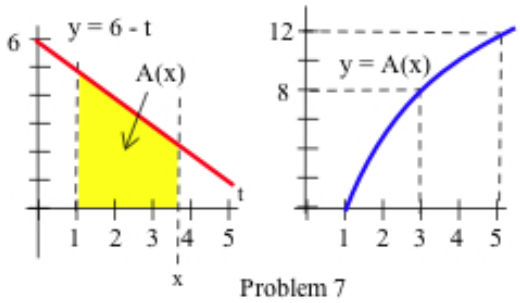
Problem 3

5. (a) see Figure
(b) $A(1) = 0$, $A(2) = 2$, $A(3) = 4$, $A(4) = 6$
(c) $A'(1) = 2$, $A'(2) = 2$, $A'(3) = 2$, $A'(4) = 2$



Problem 5

7. (a) see Figure
(b) $A(1) = 0$, $A(2) = 4.5$, $A(3) = 8$, $A(4) = 10.5$
(c) $A'(1) = 5$, $A'(2) = 4$, $A'(3) = 3$, $A'(4) = 2$



Problem 7

9. $x^2\Big|_0^3 = 9$, $x^2\Big|_1^3 = 8$, $x^2\Big|_0^1 = 1$

11. $2x^3\Big|_1^3 = 52$, $2x^3\Big|_1^2 = 14$, $2x^3\Big|_0^3 = 54$

13. $x^4\Big|_0^3 = 81$, $x^4\Big|_1^3 = 80$, $x^4\Big|_0^1 = 1$

15. $x^3\Big|_{-3}^3 = 54$, $x^3\Big|_{-3}^0 = 27$, $x^3\Big|_0^3 = 27$

17. $x^3\Big|_0^2 = 8$, $x^3\Big|_1^3 = 26$

19. (a) distance $= \int_0^{10} 2t\ dt = t^2\Big|_0^{10} = 100$ feet.

(b) Find T so $50 = \int_0^T 2t\ dt = t^2\Big|_0^T = T^2$. $T = \sqrt{50} \approx 7.07$ seconds.

21. (a) distance $= \int_0^{10} 4t^3\ dt = t^4\Big|_0^{10} = 10{,}000$ feet.

(b) $5000 = \int_0^T 4t^3\ dt = T^4$. $T = \sqrt[4]{5000} \approx 8.41$ seconds.

23. (a) velocity $= 75 - 3t^2 = 0$ when $t = 5$ seconds.(b) distance $= \int_0^5 75 - 3t^2\ dt = 75t - t^3\Big|_0^5 = 250$ feet.

(c) $125 = \int_0^T 75 - 3t^2\ dt = 75t - t^3\Big|_0^T = 75T - T^3$ so $T^3 - 75T + 125 = 0$ (solve using Newton's method or by examining the graph of $y = x^3 - 75x + 125$) and $T \approx 1.74$ seconds.

25. The total area is $\int_0^3 x^2\,dx = \frac{1}{3}x^3\Big|_0^3 = 9$.

(a) Find T so $\frac{1}{2}\cdot 9 = \frac{9}{2} = \int_0^T x^2\,dx = \frac{1}{3}x^3\Big|_0^T = \frac{1}{3}T^3$. $T = \sqrt[3]{27/2} \approx 2.38$.

(b) Find T so $\frac{1}{3}\cdot 9 = 3 = \int_0^T x^2\,dx = \frac{1}{3}x^3\Big|_0^T = \frac{1}{3}T^3$. $T = \sqrt[3]{9} \approx 2.08$.

Then find T so $\frac{2}{3}\cdot 9 = 6 = \int_0^T x^2\,dx = \frac{1}{3}x^3\Big|_0^T = \frac{1}{3}T^3$. $T = \sqrt[3]{18} \approx 2.62$.

Section 5.6

1. (a) $A(x) = x^3$. Then $A'(x) = 3x^2$, and $A'(1) = 3$, $A'(2) = 12$, and $A'(3) = 27$.

(b) $A'(x) = \mathbf{D}\left(\int_0^x 3t^2\,dt\right) = 3x^2$. $A'(1) = 3$, $A'(2) = 12$, and $A'(3) = 27$.

3. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$.

4. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$.

5. $A'(x) = 2x$ so $A'(1) = 2$, $A'(2) = 4$, $A'(3) = 6$.

6. $A'(x) = 3 - x^2$ so $A'(1) = 2$, $A'(2) = -1$, $A'(3) = -6$.

7. $A'(x) = \sin(x)$ so $A'(1) \approx 0.84$, $A'(2) \approx 0.91$, $A'(3) \approx 0.14$.

8. $A'(x) = |x - 2|$ so $A'(1) = 1$, $A'(2) = 0$, $A'(3) = 1$.

9. $A'(x) = f(x)$ so $A'(1) = 2$, $A'(2) = 1$, $A'(3) = 2$.

10. $A'(x) = f(x)$ so $A'(1) = 0$, $A'(2) = -1$, $A'(3) = 0$.

11. $A'(x) = f(x)$ so $A'(1) = 1$, $A'(2) = 2$, $A'(3) = 2$.

12. $A'(x) = f(x)$ so $A'(1) = -1$, $A'(2) = -1$, $A'(3) = 0$.

13. $F(1) - F(0) = 6 - 5 = 1$

15. $F(3) - F(1) = 9 - \frac{1}{3} = \frac{26}{3}$

17. $F(5) - F(1) \approx 1.61 - 0 = 1.61$

19. $F(3) - F(1/2) \approx 1.10 - (-0.69) = 1.79$

21. $F(\pi/2) - F(0) = 1 - 0 = 1$

23. $F(1) - F(0) \approx 0.67 - 0 = 0.67$

25. $F(7) - F(1) = \frac{2}{3}(7)^{3/2} - \frac{2}{3} \approx 11.68$

27. $F(9) - F(1) = 3 - 1 = 2$

29. $F(3) - F(-2) \approx 20.09 - 0.14 = 19.95$

31. $F(\pi/4) - F(0) = 1 - 0 = 1$

33. $F(3) - F(0) = \frac{2}{3}(10)^{3/2} - \frac{2}{3} \approx 20.42$

35. $F(x) = \frac{1}{3}x^3$. $F(2) - F(-1) = \frac{8}{3} - \left(-\frac{1}{3}\right) = 3$.

37. $F(x) = \ln(x)$. $F(e) - F(1) = 1 - 0 = 1$.

39. $F(x) = \frac{2}{3}x^{3/2}$. $F(100) - F(25) = \frac{2000}{3} - \frac{250}{3} = \frac{1750}{3} \approx 583.33$

41. $F(x) = -1/x$. $F(10) - F(1) = -0.1 - (-1) = 0.9$

43. $F(x) = e^x$. $F(1) - F(0) = e - 1 \approx 1.718$

45. $F(x) = \tan(x)$. $F(\pi/4) - F(\pi/6) \approx 1 - 0.577 = 0.423$

47. The integral goes from 3 to 3 so even without knowing an antiderivative, $\int_3^3 \sin(x)\cdot\ln(x)\,dx = 0$.

49. area $= \int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = -(-1) - (-1) = 2$ 51. area $= \int_0^{3.5} \text{INT}(x)\,dx) = 0 + 1 + 2 + \frac{1}{2}(3) = 4.5$.

53. area $= \int_0^3 (x-2)^2\,dx = \int_0^3 x^2 - 4x + 4\,dx = \frac{1}{3}x^3 - 2x^2 + 4x\Big|_0^3 = 3 - 0 = 3$.

55. $\mathbf{D(}\,A(3x)\,\mathbf{)} = \mathbf{3}\cdot\tan(3x)$, $\mathbf{D(}\,A(x^2)\,\mathbf{)} = \mathbf{2x}\cdot\tan(x^2)$, $\mathbf{D(}\,A(\sin(x))\,\mathbf{)} = \mathbf{\cos(x)}\cdot\tan(\sin(x))$

57. $\sqrt{1+5x}\cdot(5)$

59. $\sqrt{1+\sin(x)}\cdot\cos(x)$

61. $\{3(1-2x)^2 + 2\}\cdot(-2)$

63. $-\cos(3x)$

65. $\tan(x^2)\cdot 2x - \tan(x)$

67. $5\cdot\ln(x)\cdot\cos(3\cdot\ln(x))\cdot\frac{1}{x}$

Section 5.7

1. Left side $= \frac{1}{4}\,x^4\Big|_1^2 = \frac{15}{4}$. Right side $= \left\{\frac{1}{3}\,x^3\Big|_1^2 = \frac{7}{3}\right\}\cdot\left\{\frac{1}{2}\,x^2\Big|_1^2 = \frac{3}{2}\right\} = \frac{7}{2} \neq$ left side.

3. Left side $= \frac{1}{4}$. Right side $= (\frac{1}{3})\cdot(\frac{1}{2}) = \frac{1}{6} \neq$ left side.

5. $\frac{1}{3}\sin(3x) + C$

7. $-\cos(2 + e^x) + C$

9. $\tan(\sin(x)) + C$

11. $\frac{5}{2}\ln|3 + 2x| + C$

13. $-\frac{1}{3}\cos(1 + x^3) + C$

15. $\frac{1}{4}\sin(4x) + C$

17. $\frac{1}{48}(5 + x^4)^{12} + C$

19. $\ln|2 + x^3| + C$

21. $\frac{1}{2}(\ln(x))^2 + C$

23. $\frac{1}{24}(1 + 3x)^8 + C$

25. $\sec(e^x) + C$

27. $\frac{1}{3}\sin(3x)\Big|_0^{\pi/2} = -\frac{1}{3}$

29. $-\cos(2 + e^x)\Big|_0^1 = \cos(3) - \cos(2 + e) \approx -0.996$

31. $\frac{1}{18}(1 + x^3)^6\Big|_{-1}^1 = \frac{32}{9}$

33. $\frac{5}{2}\ln|3 + 2x|\Big|_0^2 = \frac{5}{2}\ln(\frac{7}{3})$

35. $-\frac{1}{3}(1 - x^2)^{3/2}\Big|_0^1 = \frac{1}{3}$

37. $\frac{2}{9}(1 + 3x)^{3/2}\Big|_0^1 = \frac{16}{9} - \frac{2}{9} = \frac{14}{9}$

39. $\frac{1}{2}\,x - \frac{1}{20}\sin(10x) + C$

41. $\frac{1}{4}\sin(2x) + C$

43. $\frac{1}{2}\,x - \frac{1}{4}\sin(2x)\Big|_0^{\pi} = \frac{\pi}{2}$

45. $\frac{1}{7}\,x^7 + \frac{3}{5}\,x^5 + x^3 + x + C$

47. $\frac{1}{2}\,e^{2x} + 2e^x + x + C$

49. $\frac{1}{6}\,x^6 + \frac{1}{4}\,x^4 + \frac{5}{3}\,x^3 + 5x + C$

51. $\frac{1}{2}\,e^{2x} + \frac{1}{4}\,e^{4x} + C$

53. $\frac{2}{7}\,x^{7/2} + \frac{6}{5}\,x^{5/2} - \frac{4}{3}\,x^{3/2} + C$

55. $3x - 3\cdot\ln|x + 1| + C$

57. $\frac{1}{2}\,x^2 - x + C$

59. (divide first) $x^2 - 11x + 7\cdot\ln|x - 1| + C$

61. (divide first) $x + 3\cdot\ln|x - 1| + C$

63. $\frac{2}{3}x^{3/2} + 8\,x^{1/2} + C$

65. (area of semicirle with radius 1) $= \frac{1}{2}\,\pi(1)^2 = \frac{\pi}{2}$

67. (area of semicirle with radius 3) $= \frac{1}{2}\,\pi(3)^2 = \frac{9}{2}\,\pi$

69. (area of rectangle) + (area of semicircle of radius 1) $= (2)(2) + \frac{1}{2}(\pi(1)^2) = 4 + \frac{\pi}{2}$

Section 5.8

1. between 11 (using left endpoints of intervals) and 6 (using right endpoints)

3. between 4 (using left endpoints of intervals) and 6 (using right endpoints)

5. Using left endpoint widths: $(0)(40)+(70)(40)+(55)(40)+(90)(40)+(130)(40)+(115)(40) = 18{,}400\ \text{ft}^2$.
Right endpoint widths (70, 55, ...) and average widths (70/2, 125/2, ...) give the same result, $18{,}400\ \text{ft}^2$.
All of these are reasonable methods for estimating the area of the island.

7. 9 9. 1 11. $\frac{1}{2} \cdot e^2 - \frac{3}{2}$ 13. $\frac{1}{32}\pi^2 + \frac{1}{4}\pi - \frac{\sqrt{2}}{2}$

15. $e^2 - 3$ 17. $3 - \frac{\pi}{4}$

19. Estimate using midpoints of unit intervals: $\frac{1}{4}\{f(1)(1)+f(2)(1)+f(3)(1)+f(4)(1)\} = \frac{19}{4}$. About $\frac{19}{4}$.

21. Estimate using midpoints of unit intervals: $\frac{1}{2}\{f(2)(1)+f(3)(1)\} = 5$. About 5 .

23. average ≈ 1 25. average $\approx \frac{11}{5}$ 27. average = 5 29. average = $\frac{13}{3}$ 31. average = $\frac{2}{\pi}$

33. (a) C = 1: average = $\frac{2}{3}$ (b) C = 9: average = 2 (c) C = 81: average = 6 (d) C = 100: average = $\frac{20}{3}$
In general, average = $\frac{2}{3}\sqrt{C}$.

35. (a) Graphically, average $\approx$ $3000 \cdot 1000 \frac{\text{calls}}{\text{hour}} = \frac{3000000}{60} \frac{\text{calls}}{\text{min}} \approx 50{,}000 \frac{\text{calls}}{\text{min}}$. (b) About $58{,}333 \frac{\text{calls}}{\text{min}}$.

37. (a) Similar to Example 5: work = 1,950 foot–pounds (b) work = 1,312.5 foot–pounds

39. (a) work = 1,200 foot–pounds (b) work = 600 foot–pounds (c) work = 400 foot–pounds

41. work = 1,275 foot–pounds

Section 5.9

1. Table #35, a = 2: $\frac{1}{2}\arctan(\frac{x}{2}) + C$ 3. Table #35, a = 5: $x^2 + \frac{2}{5}\arctan(\frac{x}{5}) + C$

5. Table #37, a = 3: $(\frac{1}{3})\ln\left|\frac{x+3}{x-3}\right| + C$ 7. Table #35, a = $\sqrt{3}$: $\frac{1}{\sqrt{3}}\arctan(\frac{x}{\sqrt{3}}) + C$

9. Table #35, a = $\sqrt{2}$: $e^x + \frac{7}{\sqrt{2}}\arctan(\frac{x}{\sqrt{2}}) + C$

11. Table #34, $a = \sqrt{5}$: $3 \cdot \arcsin\left(\frac{x}{\sqrt{5}}\right) + C$ 13. Table #35, $a = \frac{2}{5}$: $\frac{1}{10} \arctan\left(\frac{5}{2}\, x\right) + C$

15. First substitute $u = 2x$, $du = 2\,dx$. Then use Table #34 with $a = 1$: $\frac{5}{2} \cdot \arcsin(u) + C = \frac{5}{2} \cdot \arcsin(2x) + C$

17. Table #43 and substitution $u = 3x$: $\frac{2}{3} \ln\left|3x + \sqrt{1 + 9x^2}\right| + C$

19. Table #38 and substitution $u = x+1$: $(x+1) \cdot \ln|x+1| - (x+1) + C$ or $(x+1) \cdot \ln|x+1| - x + C_2$

21. Table #38 and substitution $u = 5x^2+7$: $\frac{3}{10} \left\{ (5x^2+7) \cdot \ln|5x^2+7| - (5x^2+7) \right\} + C$

23. Table #38 and substitution $u = \sin(x)$: $\sin(x) \cdot \ln|\sin(x)| - \sin(x) + C$

25. Table #44, $a = 2$: $\frac{x}{2}\sqrt{x^2+4} + \frac{1}{2}(4) \cdot \ln\left|x + \sqrt{x^2+4}\right| + C$

27. Table #44, $a = 4$: $\frac{x}{2}\sqrt{x^2+16} + \frac{1}{2}(16) \cdot \ln\left|x + \sqrt{x^2+16}\right| + C$

29. Table #35, $a = 5$: $x^2 + \frac{2}{5} \arctan\left(\frac{x}{5}\right)\Big|_1^3 = 8 + \frac{2}{5} \left\{ \arctan\left(\frac{3}{5}\right) - \text{acrtan}\left(\frac{1}{5}\right) \right\}$

31. Table #35, $a = \sqrt{3}$: $\frac{1}{\sqrt{3}} \text{acrtan}\left(\frac{x}{\sqrt{3}}\right)\Big|_{-1}^{1} = \frac{1}{\sqrt{3}} \left\{ \arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan\left(\frac{-1}{\sqrt{3}}\right) \right\}$

33. Table #34, $a = \sqrt{5}$: $3 \arcsin\left(\frac{x}{\sqrt{5}}\right)\Big|_1^2 = 3 \cdot \left\{ \arcsin\left(\frac{2}{\sqrt{5}}\right) - \arcsin\left(\frac{1}{\sqrt{5}}\right) \right\}$

35. Table #34, $a = 1/2$: $\frac{5}{2} \arcsin\left(\frac{x}{1/2}\right)\Big|_0^{0.1} = \frac{5}{2} \arcsin(0.2)$

37. $7 \cdot \ln(7) - 6$ 39. $3 \cdot \ln(3) - 2 \cdot \ln(2) - 1$ 41. $3\sqrt{18} + \frac{9}{2} \cdot \ln\left(\frac{3+\sqrt{18}}{-3+\sqrt{18}}\right)$

43. Table #19a: $\frac{-\sin^2(x) \cdot \cos(x)}{3} - \frac{2}{3} \cos(x) + C$ 45. Table #20: $\frac{\cos^4(x) \cdot \sin(x)}{5} + \frac{4}{5}$ { answer to number 44 }

47. Table #29: $x^2 \cdot \sin(x) + 2x \cdot \cos(x) - 2 \cdot \sin(x) + C$

49. Average of $\sin(x)$ on $[0,\pi]$ is $\frac{2}{\pi}$. Average of $\sin^2(x)$ on $[0,\pi]$ is $\frac{1}{2}$. $\frac{2}{\pi} > \frac{1}{2}$.

51. Using results from #50: (a) $\frac{1}{e-1}$ (b) $\frac{1}{9}\{10\cdot\ln(10)-9\} \approx \frac{14.03}{9} \approx 1.56$

(c) $\frac{1}{99}\{100\cdot\ln(100)-99\} \approx 361.52/99 \approx 3.65$ (d) $\frac{1}{199}\{200\cdot\ln(200)-199\} \approx 860.66/199 = 4.32$

53. (c) is largest.

55. approximately (a) 1.57 (b) 2.94 (c) 3.04 (d) 3.07 (e) 3.09