Computational Exercises

These exercises are selected to illustrate ways that engineers use handbook data to formulate conclusions. They are not representative of final exam questions, which will be multiple choice questions. Complete solutions are presented following the statement of the exercises.

Exercise 1

We saw in sub-subunit 3.4.3 that the relevant properties for selecting materials depend on the questions we ask. In that section, we looked at minimizing volume and minimizing weight. Frequently, a structural member can be modeled as a cantilevered beam, clamped at one end and with a force (F) applied at the other end. For a beam of square cross-section, the deflection (δ) of the forced end is given by:

$$\delta = \left(\frac{1}{E}\right)\left(\frac{4L^3}{w^4}\right)(F)$$

The first bracket contains the material property Modulus of Elasticity (E), the second contains geometry factors of beam length (L) and cross-section width (w), and the third bracket contains the experimental conditions, in this case the applied force (F).

The mass of the beam is computed as:

$$m = (\rho)(w^2 L)$$

with ρ as the mass density of the material.

Combining these equations to eliminate w yields:

$$m = \left(\frac{\rho}{\sqrt{E}}\right)\left(4L^5\right)^{1/2}\left(\frac{F}{\delta}\right)^{1/2}.$$

1. Use the data in the table below to order the materials in terms of minimum weight, considering that the length (*L*) and stiffness (*F/δ*) are specified as design parameters.

2. Also included in the table is an estimate of relative cost per unit mass. Use this data to reorder the materials in terms of minimum cost.

	SG	*E* (GPa)	relative cost*
steel	7.8	200	100
aluminum alloy	2.7	75	400
GFRP	1.8	50	1000
CFRP	1.6	150	20,000
concrete	2.5	40	50
wood	0.6	10	70

* normalized with (steel = 100)

GFRP – glass fiber reinforced polymer composite
CRFP – carbon fiber reinforced polymer composite

Answer:

Ordered by weight:

rank	least weight
1	CFRP
2	wood
3	GFRP
4	aluminum alloy
5	concrete
6	steel

Ordered by cost:

rank	least cost
1	wood
2	concrete
3	steel
4	aluminum alloy
5	GFRP
6	CFRP

Note: The assumption of a square cross-section is specialized. As illustrated in sub-subunit 3.3.4, we can affect the ordering by considering different cross-sectional shapes, most commonly I-beams.

Solution to Exercise 1

Computations using the numbers in the statement of the problem:

	$(SG)/\sqrt{(E)}$	relative cost
steel	0.55	55
aluminum alloy	0.31	125
GFRP	0.25	255
CFRP	0.13	2610
concrete	0.40	20
wood	0.19	13

Exercise 2

In subunit 1.2 of Unit 1, we identified plastic deformation as shear deformation within the interior of grains. We will see in sub-subunit 4.2.1 of Unit 4 that the mechanism of deformation is the movement of *dislocation lines*, or lines of defective atomic arrangement that are created in very large numbers by plastic deformation.

Advanced materials courses characterize dislocations by a displacement *Burgers vector* (b), which we will take as twice the ionic radius in magnitude. Because of less dense atomic packing along a dislocation line, the introduction of dislocations slightly increases the volume of a material. This dilation is estimated as:

$\Delta = b^2/4$,

which has the units of m^2 and can be interpreted as meters cubed of volume increase per meter of dislocation formed. Estimate the change in the density of copper if (1×10^{14} m^2) of dislocations are introduced by cold working. (This number can in turn be interpreted as meters of dislocation per cubic meter of material.) For atomic radius, use:

$R = 1.28$ Angstroms (1.28×10^{-10} m)

Answer:

$\rho_f = \rho_0\,[1 - (1.6\times10^{-6})]$

with ρ_f being the final density and ρ_0 being the initial density. Even with this large amount of dislocation content, the density change is much too small to be detectable with common laboratory equipment. Direct evidence of dislocations comes from the electron microscope.

Solution to Exercise 2

Numerically, the volume dilation per length of dislocation is:

$\Delta = [2(1.28\times10^{-10})]^2/4$
$= 1.6\times10^{-20}\ \text{m}^2.$

Consider L_D to be the total length of dislocation lines introduced. The change in volume can now be calculated as:

$V_f = V_0 + L_D\,\Delta.$

The dislocation density is defined to be L_D/V_f. Assuming that $V_f \approx V_0$, we now have:

$V_f = V_0\,[1 + (1\times10^{14})(1.6\times10^{-20})]$
$= V_0\,[1 + (1.6\times10^{-6})].$

and we see that our assumption that $V_f \approx V_0$ is very good. Dividing this expression into the sample mass to obtain density gives the answer, remembering that for
$x < 1$,

$$\frac{1}{(1+x)} \approx (1-x).$$