Computational Exercises

These exercises are selected to illustrate ways that engineers use handbook data to formulate conclusions. They are not representative of final exam questions, which will be multiple choice questions. Complete solutions are presented following the statement of the exercises.

Exercise 1

A steel alloy has a yield stress of 750 MPa, and a fracture toughness of 50 MPa•√m. An aluminum alloy has a yield stress of 150 MPa and a fracture toughness of 20 MPa•√m. Each is examined by X-ray methods and found to contain no detectable cracks. The equipment can detect single edge cracks of *a* = 3.0 mm or greater. Assuming that the metals contain cracks on the limit of detection, determine the mode of failure for each material.

Answer: Steel is predicted to fail by fast fracture; aluminum is predicted to fail by plastic yielding.

Note: Because material properties are sensitive to alloy content and heat treatment, we cannot generalize about the failure modes of steel and aluminum from this calculation.

Solution to Exercise 1

Calculate the stress required to cause fast fracture for each alloy.

For the steel:

$$\sigma_c = \frac{50}{\sqrt{\pi(3\times10^{-3})}} \;.$$
$$= 515\,MPa$$

This is less than the yield stress of 750 MPa, so we conclude that failure is by fast fracture.

For the aluminum:

$$\sigma_c = \frac{20}{\sqrt{\pi(3\times10^{-3})}} \;.$$
$$= 206\,MPa$$

This is greater than the yield stress of 150 MPa, so we conclude that failure is by plastic yielding.

Exercise 2

At a temperature of 160 ºC, it requires 80 seconds to force a thermosetting polymer into an injection mold. At 180 ºC, this time is reduced to 30 seconds. Assuming that the viscosity of the polymer follows an Arrhenius law temperature dependence, calculate the time predicted if the temperature is further raised to 230 ºC.

Answer: At a temperature of 230 ºC, it is predicted that 3.6 seconds will be required to force the polymer into the mold.

Note: To have confidence in an activation energy calculation, we would generally wish to have more than just two points. A much better approach is to fit a least squares linear regression line to several data points.

Solution to Exercise 2

The Arrhenius law temperature dependence is

$$\begin{aligned} (rate) &= 1/(time) \\ &= A \exp(-Q/RT) \end{aligned}$$

See sub-subunit 1.6.2.1 for a definition of terms. We can eliminate the constant A by considering a ratio of the given times. Remember that temperature must be expressed in kelvin, and also that:

$$\ln(e^x) = x$$

Thus,

$$\ln\left(\frac{80}{30}\right) = -\left(\frac{Q}{R}\right)\left(\frac{1}{453} - \frac{1}{433}\right)$$

$$\frac{Q}{R} = 9620\,K$$

and the time we seek is computed as

$$\begin{aligned} t &= (80)\exp\left[-(9620)\left(\frac{1}{433} - \frac{1}{503}\right)\right]. \\ &= 3.6\,s \end{aligned}$$

Alternatively, this can be calculated from

$$t = (30)\exp\left[-(9620)\left(\frac{1}{453} - \frac{1}{503}\right)\right].$$
$$= 3.6\, s$$