

Elastic Constants

For rigid solids (metals, plastics, and ceramics), we observe that the initial deformation is totally elastic, with the stresses being linearly dependent on the strains, if the strains do not exceed a few tenths of a percent. Six stresses dependent on six strains suggests a set of linear equations with $6^2 = 36$ coefficients. The symmetry of the stress and strain definitions reduces these to 21 independent elastic constants for the most general case.

Fortunately, we almost never deal with the most general case. Symmetries of atomic or molecular spatial arrangements reduce the number of independent elastic constants further. The most common assumption is that our material is *isotropic* – the same in all directions. For this case, there are only two independent elastic constants!

There are three common elastic constants found in most reference books. Since only two are independent, there is an equation that, in theory, permits the calculation of the third when the other two have been measured experimentally. Two elastic constants are defined from the normal stresses and strains:

- Elastic Modulus (E)

$$E = \frac{S}{e}$$

This is also referred to as Young's Modulus, after the English researcher Thomas Young (1773–1829).

- Poisson's Ratio (ν , the Greek letter nu)

$$\nu = -\frac{e_x}{e_y}$$

Subscript y refers to strain along the axis of applied force, and subscript x refers to strain along a direction transverse to the applied force. Experimentally, the two strains will have opposite signs, so including the negative sign in the definition results in a positive value for Poisson's ratio.

The remaining common elastic constant is defined from the shear stress and strain:

- Shear Modulus (G, also referred to as the Modulus of Rigidity)

$$G = \frac{t}{g}$$

The equation relating these three elastic constants for isotropic materials is:

$$G = \frac{E}{2(1 + \nu)}$$

If ν is experimentally measured, often by placing strain gauges on a tensile sample, then this equation is a useful relationship between E and G . We find for many metals that $\nu \approx 1/3$ and thus $G \approx 3/8 E$. However, the above equation is not useful for calculating ν . Rearranging this equation, we have:

$$\nu = \frac{E - 2G}{2G}$$

Our calculation involves a small difference between two large numbers, and rarely are E and G known with sufficient accuracy to accurately compute ν .

For this course we shall assume that our materials are isotropic and use E to compare the elastic deformation behavior of different materials. Be aware, though, that this is not always a valid assumption. A common exception is metals that have been formed by a rolling process. Usually, properties are different along the roll direction than across it, a condition referred to as *preferred orientation*.

Omitted from this discussion has been rubbery materials (elastomers). These materials can undergo elastic deformations that are orders of magnitude greater than the rigid solids. Also with these materials, the relationships between stress and strain are usually non-linear.