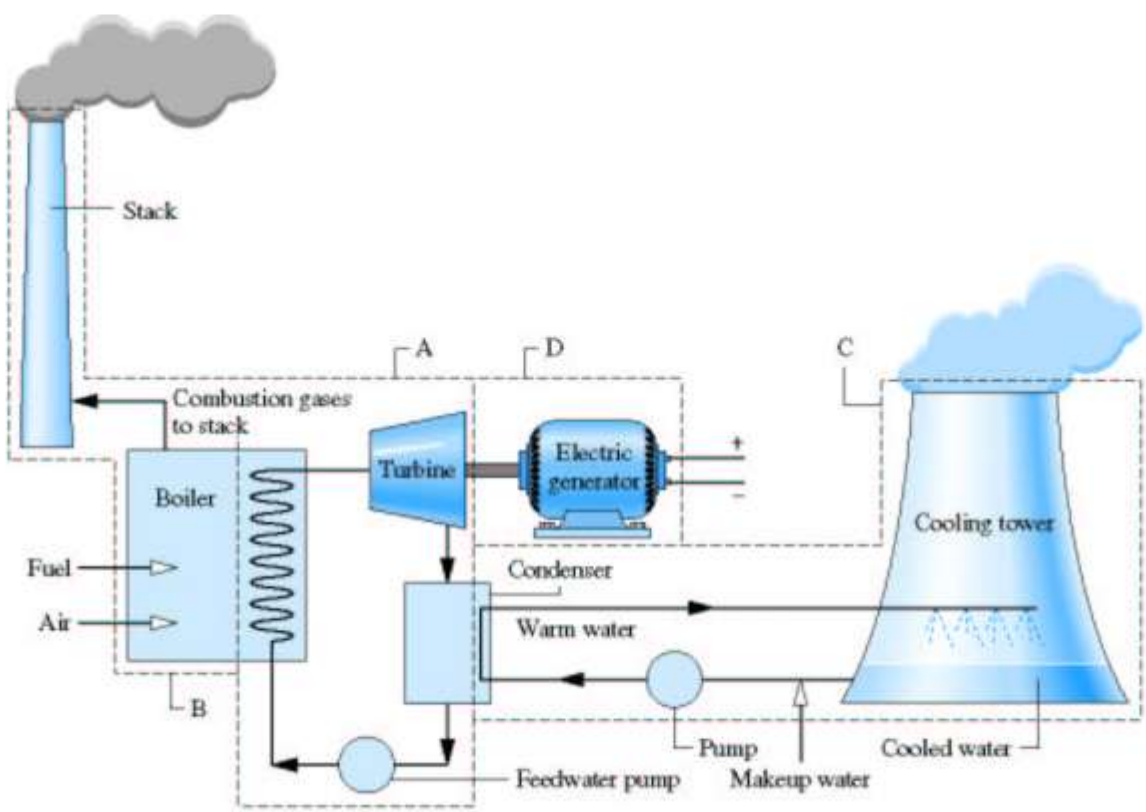
Vapor Power Systems

Power plants work on a cycle that produces net work from a fossil fuel (natural gas, oil, coal) nuclear, or solar input.

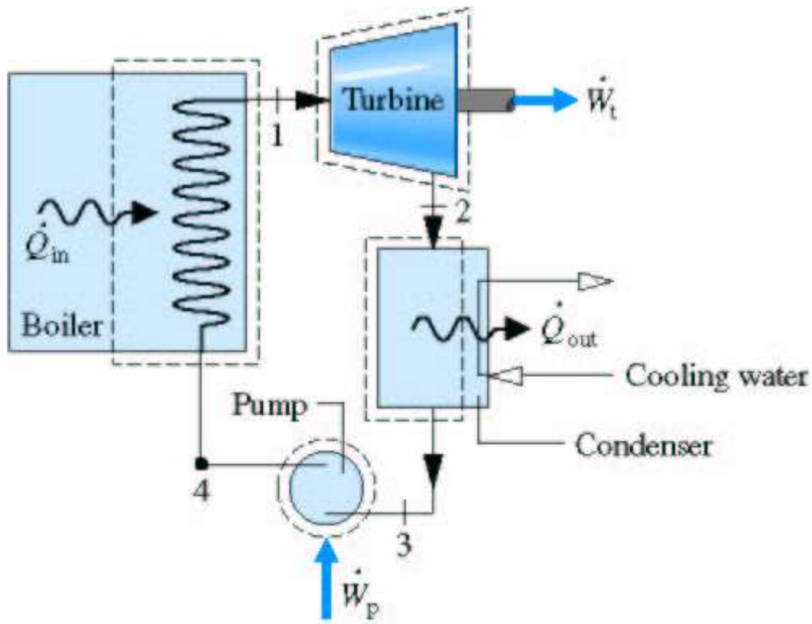
For **Vapor power plants** the working fluid, typically water, is alternately vaporized and condensed.

Consider the following Simple Vapor Power Plant



Consider subsystem A, each unit of mass periodically undergoes a thermodynamic cycle as the working fluid circulates through the four interconnected components

For the purpose of analyzing the performance of the system, the following cycle describes the basic system



Consider each process separately applying conservation of energy

For steady-state, neglecting KE and PE effects, conservation of energy applied to a CV yields

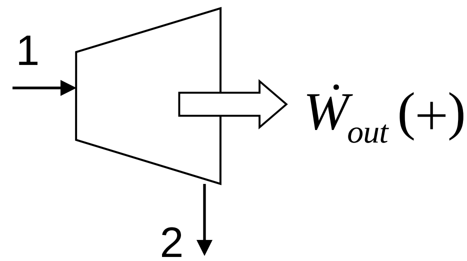
$$\frac{1}{\dot{m}}\cancel{\frac{dE}{dt}} = \frac{\dot{Q}_{CV}}{\dot{m}} - \frac{\dot{W}_{CV}}{\dot{m}} + (h_{in} - h_{out}) + 1/2(\cancel{V_{in}^2 - V_{out}^2}) + g(\cancel{z_{in} - z_{out}})$$

$$0 = \frac{\dot{Q}_{CV}}{\dot{m}} - \frac{\dot{W}_{CV}}{\dot{m}} + (h_{in} - h_{out})$$

1→2 Turbine (adiabatic expansion)

$$0 = \cancel{\frac{\dot{Q}}{\dot{m}}} - \frac{\dot{W}_{out}}{\dot{m}} + (h_1 - h_2)$$

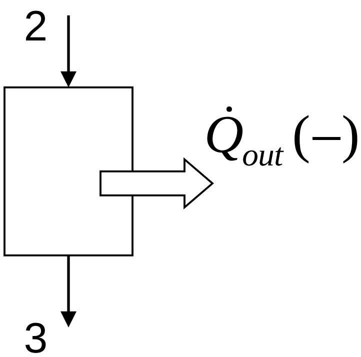
$$\boxed{w_{out} = \frac{\dot{W}_{out}}{\dot{m}} = (h_1 - h_2)}$$



2→3 Condenser (no work)

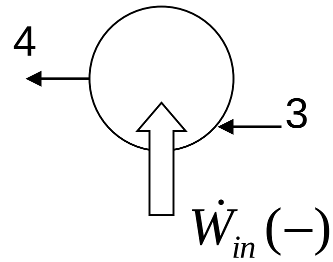
$$0 = \frac{-\dot{Q}_{out}}{\dot{m}} - \cancel{\frac{\dot{W}}{\dot{m}}} + (h_2 - h_3)$$

$$\boxed{q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = (h_2 - h_3)}$$



3→4 Pump (Adiabatic)

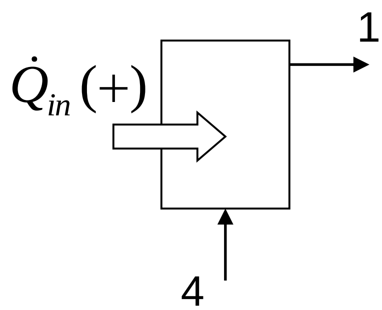
$$0 = \frac{\cancel{\dot{Q}}}{\dot{m}} - \frac{-\dot{W}_{in}}{\dot{m}} + (h_3 - h_4)$$



$$\boxed{w_{in} = \frac{\dot{W}_{in}}{\dot{m}} = (h_4 - h_3)}$$

4→1 Steam Generator (no work)

$$0 = \frac{\dot{Q}_{in}}{\dot{m}} - \frac{\cancel{\dot{W}}}{\dot{m}} + (h_4 - h_1)$$



$$\boxed{q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4)}$$

Rankine Cycle Thermal Efficiency

$$\eta = \frac{\text{net work out}}{\text{heat input}} = \frac{\left(\dot{W}_{out} / \dot{m}\right) - \left(\dot{W}_{in} / \dot{m}\right)}{\dot{Q}_{in} / \dot{m}} = \frac{w_{out} - w_{in}}{q_{in}}$$

$$\boxed{\eta_{Rankine} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}}$$

Back Work Ratio (bwr)

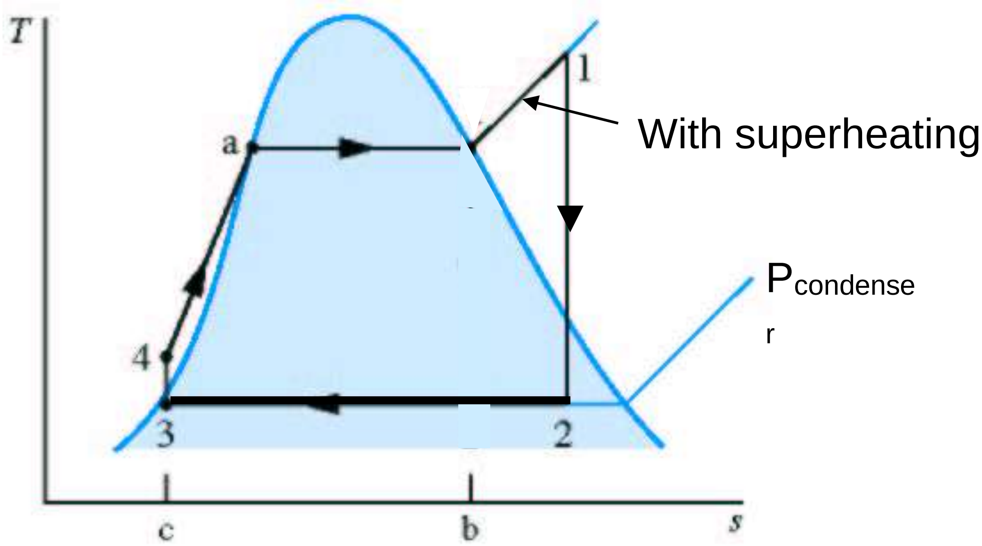
$$bwr = \frac{\text{work input (pump)}}{\text{work output (turbine)}} = \frac{\dot{W}_{in} / \dot{m}}{\dot{W}_{out} / \dot{m}} = \frac{w_{in}}{w_{out}}$$

$$\boxed{bwr = \frac{h_4 - h_3}{h_1 - h_2}}$$

Ideal Rankine Cycle - no irreversibilities present in any of the processes: no fluid friction so no pressure drop, and no heat loss to surroundings

1. Steam generation occurs at constant pressure 4→1
2. Isentropic expansion in the turbine 1→2
3. Condensation occurs at constant pressure 2→3
4. Isentropic compression in the pump 3→4

P_{boiler}



Note: For an ideal cycle no irreversibilities present so the pump work can be evaluated by

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = -\int_3^4 v\,dP$$

if the working fluid entering the pump at state 3 is pure liquid, then

$$w_{in} = \left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{\text{int}\\ rev}} = \int_3^4 v\,dP = v_3\left(P_4 - P_3\right)$$

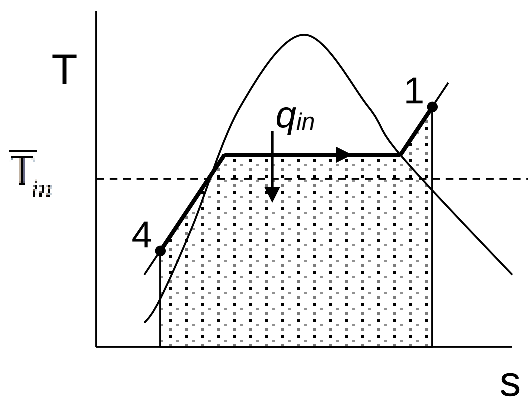
The negative sign has been dropped to be consistent with previous use of w_{in}

Factors Affecting Cycle Efficiency

$$\eta = \frac{w_{out} - w_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

Recall: for a reversible heat addition process $q = \int T ds$

Consider q_{in} at the boiler and q_{out} at the condenser

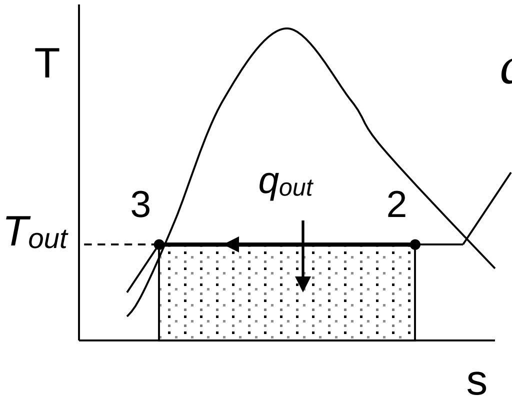


$$q_{in} = q_{4\to1} = \int_4^1 T ds$$
$$= \text{shaded area}$$

Define mean temperature for process 4 → 1

$$\overline{T}_{in} = \frac{\int_4^1 T ds}{s_1 - s_4}$$

$$\therefore q_{in} = \int_4^1 T ds = \overline{T}_{in} \int_4^1 ds = \overline{T}_{in}\left(s_1 - s_4\right)$$



$$q_{out} = |q_{2\to3}| = \left|\int_2^3 Tds\right|$$
$$= T_{out}(s_2 - s_3)$$
$$= \text{shaded area}$$

Noting $s_2 - s_3 = s_1 - s_4$, the Ideal Rankine cycle thermal efficiency is

$$\eta_{\substack{Ideal\\Rankine}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_{out}\cancel{(s_2 - s_3)}}{\overline{T}_{in}\cancel{(s_1 - s_4)}} = 1 - \frac{T_{out}}{\overline{T}_{in}}$$

Note: this is identical to the Carnot Engine efficiency which is also a reversible cycle

The back work ratio is

$$bwr_{\substack{Ideal\\Rankine}} = \frac{w_{in}}{w_{out}} = \frac{v_3(P_4 - P_3)}{(h_1 - h_{2s})}$$

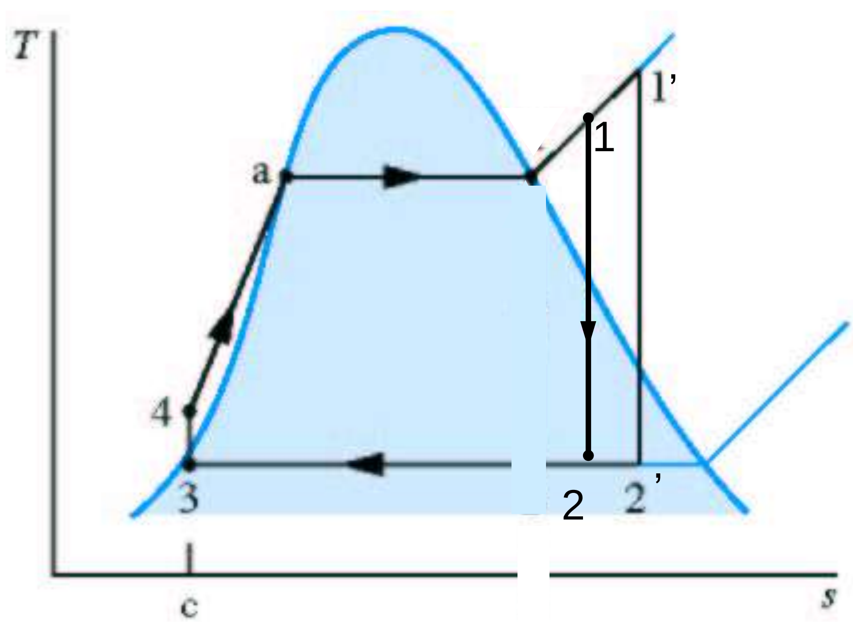
Increase Rankine Cycle Efficiency

$$\eta_{\substack{Ideal \\ Rankine}} = 1 - \frac{T_{out}}{\bar{T}_{in}}$$

Cycle efficiency can be improved by either:

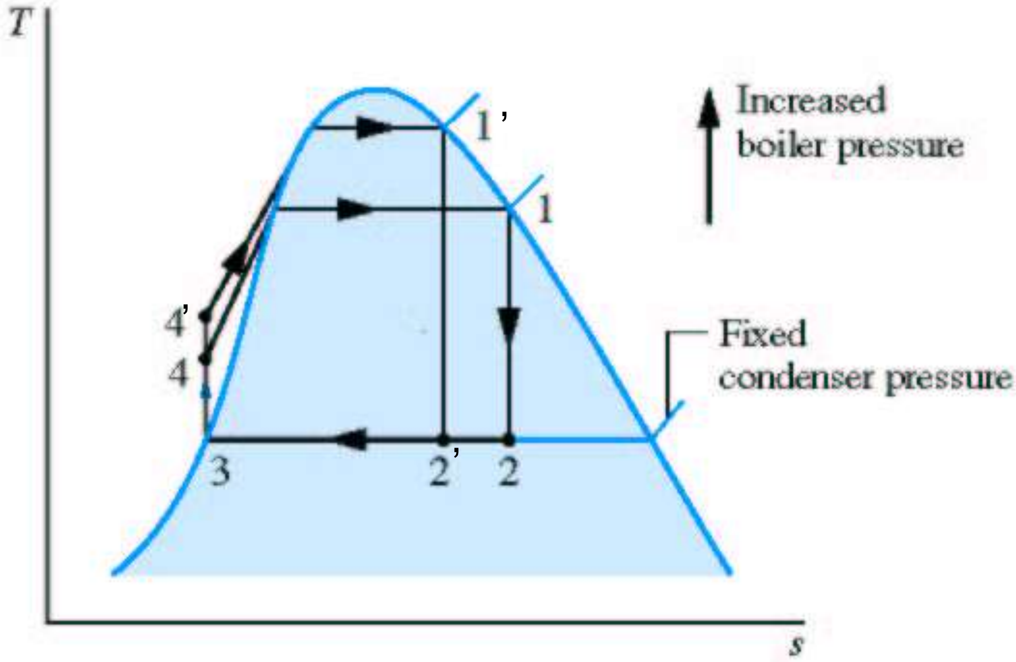
- increasing the average temperature during heat addition ($\bar{T}_{in}$)
- decreasing the condenser temperature (T_{out})

Increase the amount of superheat (4→1')



Amount of superheating is limited by metallurgical considerations of the turbine (T_1 < 670C)
Added benefit is that the quality of the steam at the turbine exit is higher

Increase boiler pressure (4 → 1’)

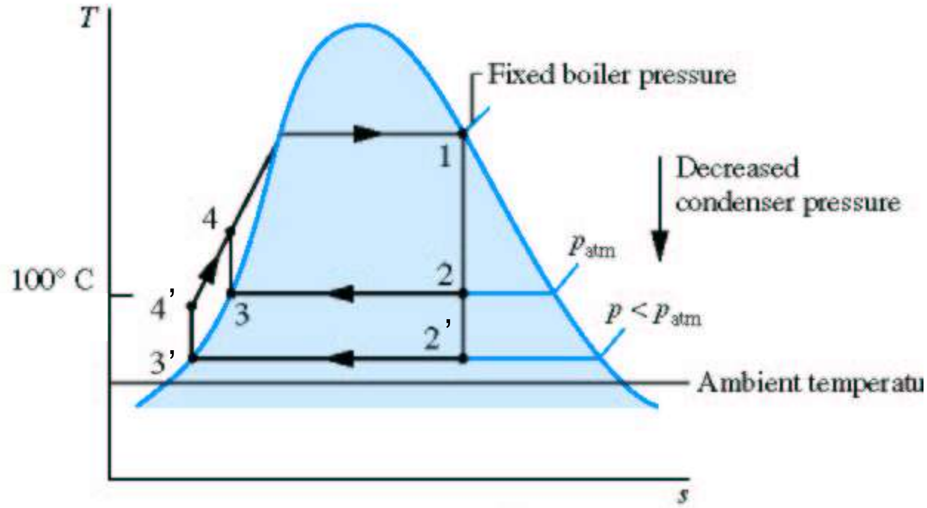


Disadvantages:

- Requires more robust equipment

- Vapor quality at 2’ lower than at 2

Decrease Condenser Pressure (2’ → 3’)



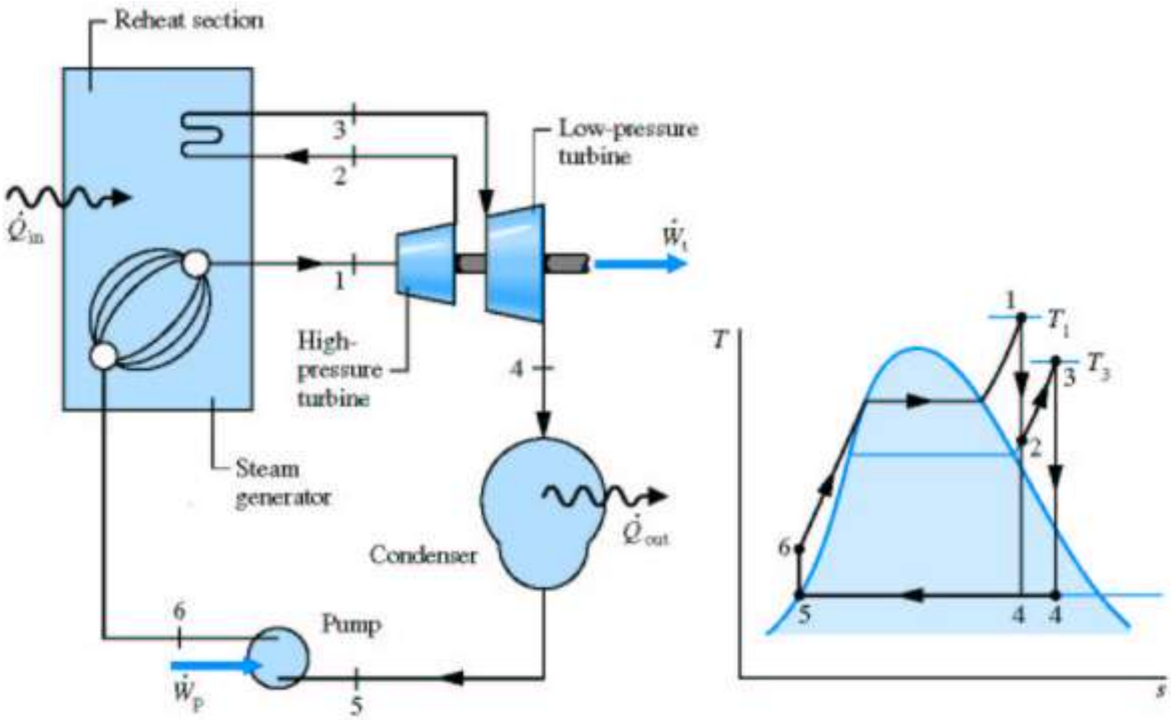
T_{out} is limited to the temperature of the cooling medium (e.g., lake at 15C need 10C temperature difference for heat transfer so T_{out} >25C)

Disadvantages:

- Note: for water P_{sat}(25C)= 3.2 kPa lower than atmospheric, possible air leakage into lines

- Vapor quality lower at lower pressure not good for turbine

The most common method to increase the cycle thermal efficiency is to use a two-stage turbine and **reheat** the steam in the boiler after the first stage



$$\eta = \frac{\text{net work out}}{\text{heat input}} = \frac{w_{out} - w_{in}}{q_{in}} = \frac{(w_{1\to2} + w_{3\to4}) - w_{5\to6}}{(q_{6\to1} + q_{2\to3})}$$

$$\boxed{\eta_{\substack{Rankine \\ w/reheat}} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}}$$