3.7 RELATED RATES: An Application of Derivatives

In the next several sections we'll look at more uses of derivatives. Probably no single application will be of interest or use to everyone, but at least some of them should be useful to you. Applications also reinforce what you have been practicing; they require that you recall what a derivative means and use the techniques covered in the last several sections. Most people gain a deeper understanding and appreciation of a tool as they use it, and differentiation is both a powerful concept and a useful tool.

The Derivative As A Rate of Change

In Section 2.1, several interpretations were given for the derivative of a function. Here we will examine how the "rate of change of a function" interpretation can be used. If several variables or quantities are related to each other and some of the variables are changing at a known rate, then we can use derivatives to determine how rapidly the other variables must be changing.

Example 1: Suppose we know that the radius of a circle is increasing at a rate of 10 feet each second (Fig. 1), and we want to know how fast the area of the circle is increasing when the radius is 5 feet. What can we do?

Solution: We could get an approximate answer by calculating the area of the circle when the radius is 5 feet \( A = \pi r^2 = \pi (5 \text{ feet})^2 \approx 78.6 \text{ feet}^2 \) and 1 second later when the radius is 10 feet larger than before \( A = \pi (15 \text{ feet})^2 \approx 706.9 \text{ feet}^2 \) and then finding \( \frac{\Delta A}{\Delta t} = \frac{706.9 \text{ ft}^2 - 78.6 \text{ ft}^2}{1 \text{ sec}} = 628.3 \text{ ft}^2/\text{sec} \). This approximate answer represents the average change in area during the 1 second period when the radius increased from 5 feet to 15 feet. It is the slope of the secant line through the points \( P \) and \( Q \) in Fig. 2, and it is clearly not a very good approximation of the instantaneous rate of change of the area, the slope of the tangent line at the point \( P \).
We could get a better approximation by calculating \( \Delta A/\Delta t \) over a shorter time interval, say \( \Delta t = 0.1 \) seconds. Then the original area was \( 78.6 \text{ ft}^2 \), the new area is \( A = \pi(6 \text{ feet})^2 \approx 113.1 \text{ ft}^2 \) (Why is the new radius 6 feet?) so \( \Delta A/\Delta t = (113.1 \text{ ft}^2 - 78.6 \text{ ft}^2)/(0.1 \text{ sec}) = 345 \text{ ft}^2/\text{sec} \). This is the slope of the secant line through the point \( P \) and \( R \) in Fig. 3, and it is a much better approximation of the slope of the tangent line at \( P \), but it is still only an approximation. Using derivatives, we can get an exact answer without doing very much work.

We know that the two variables in this problem, the radius \( r \) and the area \( A \), are related to each other by the formula \( A = \pi r^2 \), and we know that both \( r \) and \( A \) are changing over time so each of them is a function of an additional variable \( t = \text{time} \). We will continue to write the radius and area variables as \( r \) and \( A \), but it is important to remember that each of them is really a function of \( t \), \( r = r(t) \) and \( A = A(t) \). The statement that "the radius is increasing at a rate of 10 feet each second" can be translated into a mathematical statement about the rate of change, the derivative of \( r \) with respect to time: \( \frac{dr}{dt} = 10 \text{ ft/sec} \). The question about the rate of change of the area is a question about \( \frac{dA}{dt} \). Collecting all of this information, we have

**Variables:** \( r(t) = \text{radius at time } t \), \( A(t) = \text{area at time } t \)

**We Know:** \( r = 5 \text{ feet} \) and \( \frac{dr(t)}{dt} = 10 \text{ ft/sec} \).

**We Want:** \( \frac{dA(t)}{dt} \) when \( r = 5 \)

**Connecting Equation:** \( A = \pi r^2 \) or \( A(t) = \pi r^2(t) \).

Finally, we are ready to find \( \frac{dA}{dt} \) — we just need to differentiate each side of the equation \( A = \pi r^2 \) with respect to the independent variable \( t \).
\[
\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \frac{d}{dt}r^2 = \frac{d}{dt}(2\pi r) = \pi 2r \frac{dr}{dt}.
\]

The last piece, \( \frac{dr}{dt} \), appears in the derivative because \( r \) is a function of \( t \) and we must use the differentiation rule for a function to a power (or the Chain Rule):

\[
\frac{d}{dt} f^n(t) = n f^{n-1}(t) \frac{df(t)}{dt}.
\]

We know from the problem that \( \frac{dr}{dt} = 10 \text{ ft/sec} \) so \( \frac{dA}{dt} = \pi 2r \frac{dr}{dt} = \pi 2r (10 \text{ ft/s}) = 20\pi \text{ ft/s} \). This answer tells us that the rate of increase of the area of the circle, \( \frac{dA}{dt} \), depends on the value of the radius \( r \) as well as on the value of \( \frac{dr}{dt} \). Since \( r = 5 \text{ feet} \), the area of the circle will be increasing at a rate of

\[
\frac{dA}{dt} = 20\pi \text{ ft/s} = 20\pi(5 \text{ feet}) \text{ ft/s} = 100\pi \text{ ft}^2/\text{s} \approx 314.2 \text{ square feet per second}.
\]

The key steps in finding the exact rate of change of the area of the circle were to:

- write the known information in a mathematical form, expressing rates of change as derivatives (\( r = 5 \text{ feet} \) and \( \frac{dr}{dt} = 10 \text{ ft/sec} \))
- write the question in a mathematical form (\( \frac{dA}{dt} = ? \))
- find an equation connecting or relating the variables (\( A = \pi r^2 \))
- differentiate both sides of the equation relating the variables, remembering that the variables are functions of \( t \) (\( \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \))
- put all of the known values into the equation in the previous step and solve for the desired part in the resulting equation (\( \frac{dA}{dt} = 2\pi(5 \text{ ft})(10 \text{ ft/sec}) = 314.2 \text{ ft}^2/\text{sec} \))

**Example 2:** Divers lives depend on understanding situations involving related rates. In water, the pressure at a depth of \( x \) feet is approximately \( P(x) = 15(1 + \frac{x}{33}) \) pounds per square inch (compared to approximately 15 pounds per square inch at sea level = \( P(0) \)). Volume is inversely proportional to the pressure, \( v = k/p \), so doubling the pressure will result in half the original volume. Remember that volume is a function of the pressure: \( v = v(p) \).

(a) Suppose a diver's lungs, at a depth of 66 feet, contained 1 cubic foot of air, and the diver ascended to the surface without releasing any air, what would happen?

(b) If a diver started at a depth of 66 feet and ascended at a rate of 2 feet per second, how fast would the pressure be changing?
(Dives deeper than 50 feet also involve a risk of the "bends" or decompression sickness if the ascent is too rapid. Tables are available which show the safe rates of ascent from different depths.)

Solution: (a) The diver would risk getting ruptured lungs. The 1 cubic foot of air at a depth of 66 feet would be at a pressure of \( P(66) = 15(1 + \frac{66}{33}) = 45 \) pounds per square inch (psi). Since the pressure at sea level, \( P(0) = 15 \) psi, is only 1/3 as great, each cubic foot of air would expand to 3 cubic feet, and the diver's lungs would be in danger. Divers are taught to release air as they ascend to avoid this danger.

(b) The diver is ascending at a rate of 2 feet/second so the rate of change of the diver's depth \( x(t) \) is \( \frac{dx}{dt} = -2 \) ft/s. The pressure, \( P = 15\left(1 + \frac{x}{33}\right) = 15 + \frac{15}{33}x \), is a function of \( x \) (or \( x(t) \)) so 

\[
\frac{dP}{dx} \cdot \frac{dx}{dt} = \frac{15}{33} \text{ psi/ft} \cdot (-2 \text{ ft/sec}) = -\frac{30}{33} \text{ psi/sec} \approx -0.91 \text{ psi/sec}.
\]

Example 3: The height of a cylinder is increasing at 7 meters per second and the radius is increasing at 3 meters per second. How fast is the volume changing when the cylinder is 5 meters high and has a radius of 6 meters? (Fig. 4)

Solution: First we need to translate our known information into a mathematical format. The height and radius are given: \( h = \text{height} = 5 \) m and \( r = \text{radius} = 6 \) m. We are also told how fast \( h \) and \( r \) are changing: \( \frac{dh}{dt} = 7 \) m/s and \( \frac{dr}{dt} = 3 \) m/s. Finally, we are asked to find \( \frac{dV}{dt} \), and we should expect the units of \( \frac{dV}{dt} \) to be the same as \( \Delta V/\Delta t \) which are \( \text{m}^3/\text{s} \).

Variables: \( h(t) = \text{height at time } t \) , \( r(t) = \text{radius at } t \) , \( V(t) = \text{volume at } t \).

Know: \( h = 5 \) m , \( \frac{dh(t)}{dt} = 7 \) m/s , \( r = 6 \) m , \( \frac{dr(t)}{dt} = 3 \) m/s.

Want: \( \frac{dV(t)}{dt} \)

We also need an equation which relates the variables \( h \) , \( r \) and \( V \) (all of which are functions of time \( t \)) to each other:

Connecting Equation: \( V = \pi r^2 h \) or \( V(t) = \pi r^2(t) h(t) \)

Then, differentiating each side of this equation with respect to \( t \) (remembering that \( h \) , \( r \) and \( V \) are functions), we have
\[
\frac{dV}{dt} = \frac{d(\pi r^2h)}{dt} = \pi \left( r^2 \frac{dh}{dt} + h \frac{dr^2}{dt} \right) \text{ by the Product Rule}
\]
\[
= \pi \left( r^2 \frac{dh}{dt} + h(2r) \frac{dr}{dt} \right) \text{ by the Power Rule for functions.}
\]

The rest is just substituting values and doing some arithmetic:
\[
= \pi \left\{ (6 \text{ m})^2 (7 \text{ m/s}) + (5 \text{ m})2(6 \text{ m})(3 \text{ m/s}) \right\}
= \pi \left\{ 252 \text{ m}^3/\text{s} + 180 \text{ m}^3/\text{s} \right\}
= 432\pi \text{ m}^3/\text{s} \approx 1357.2 \text{ m}^3/\text{s}.
\]

The volume of the cylinder is increasing at a rate of 1357.2 cubic meters per second. (It is always encouraging when the units of our answer are the ones we expect.)

**Practice 1:** How fast is the surface area of the cylinder changing in the previous example? (Assume that \( h, \ r, \ \frac{dh}{dt}, \ \text{and} \ \frac{dr}{dt} \) have the same values as in the example and use Fig. 5 to help you determine an equation relating the surface area of the cylinder to the variables \( h \) and \( r \). The cylinder has a top and bottom.)

**Practice 2:** How fast is the volume of the cylinder in the previous example changing if the radius is decreasing at a rate of 3 meters per second? (The height, radius and rate of change of the height are the same as in the previous example: 5 m, 6 m and 7 m/s respectively.)

Usually, the most difficult part of Related Rate problems is to find an equation which relates or connects all of the variables. In the previous problems, the relating equations required a knowledge of geometry and formulas for areas and volumes (or knowing where to find them). Other Related Rates problems may require other information about similar triangles, the Pythagorean formula, or trigonometry — it depends on the problem.

It is a good idea, a very good idea, to draw a picture of the physical situation whenever possible. It is also a good idea, particularly if the problem is very important (your next raise depends on getting the right answer), to calculate at least one approximate answer as a check of your exact answer.

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**Source URL:** [http://scidiv.bellevuecollege.edu/dh/Calculus_all/Calculus_all.html](http://scidiv.bellevuecollege.edu/dh/Calculus_all/Calculus_all.html)

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Attributed to: Dale Hoffman
Example 4: Water is flowing into a conical tank at a rate of 5 m³/s. If the radius of the top of the cone is 2 m (see Fig. 6), the height is 7 m, and the depth of the water is 4 m, then how fast is the water level rising?

Solution: Let's define our variables to be h = height (or depth) of the water in the cone and V = the volume of the water in the cone. Both h and V are changing, and both of them are functions of time t. We are told in the problem that h = 4 m and \( \frac{dV}{dt} = 5 \text{ m}^3/\text{s} \), and we are asked to find \( \frac{dh}{dt} \). We expect that the units of \( \frac{dh}{dt} \) will be the same as \( \frac{\Delta h}{\Delta t} \) which are meters/second.

| Variables: | h(t) = height at t, r(t) = radius of the top surface of the water at t, V(t) = volume of water at time t |
| Know: | h = 4 m, \( \frac{dV(t)}{dt} = 5 \text{ m}^3/\text{s} \) |
| Want: | \( \frac{dh(t)}{dt} \) |

Unfortunately, the equation for the volume of a cone, \( V = \frac{1}{3} \pi r^2 h \), also involves an additional variable r, the radius of the cone at the top of the water. This is a situation in which the picture can be a great help by suggesting that we have a pair of similar triangles so \( \frac{r}{h} = \frac{\text{top radius}}{\text{total height}} = \frac{2}{7} \) and \( r = \frac{2}{7} h \). Then we can rewrite the volume of the cone of water, \( V = \frac{1}{3} \pi r^2 h \), as a function of the single variable h:

**Connecting Equation:** \( V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{2}{7} h\right)^2 h = \frac{4}{147} \pi h^3 \).

The rest of the solution is straightforward.

\[
\frac{dV}{dt} = \frac{d}{dt} \left( \frac{4}{147} \pi h^3 \right) = \frac{4}{147} \pi \frac{dh^3}{dt} = \frac{4}{147} \pi 3h^2 \frac{dh}{dt} \quad \text{remember, h is a function of t}
\]

\[
= \frac{4}{147} \pi 3(4 \text{ m}^2) \frac{dh}{dt} \approx (4.10 \text{ m}^2) \frac{dh}{dt}.
\]

We know that \( \frac{dV}{dt} = 5 \text{ m}^3/\text{s} \) and \( \frac{dV}{dt} = (4.10 \text{ m}^2) \frac{dh}{dt} \) so it is easy to solve for

\[
\frac{dh}{dt} = \frac{\frac{dV}{dt}}{(4.10 \text{ m}^2)} = \frac{5 \text{ m}^3/\text{s}}{4.10 \text{ m}^2} \approx 1.22 \text{ m/s}.
\]
This example was a little more difficult than the others because we needed to use similar triangles to get an equation relating $V$ to $h$ and because we eventually needed to do a little arithmetic to solve for $dh/dt$.

**Practice 3:** A rainbow trout has taken the fly at the end of a 60 foot line, and the line is being reeled in at a rate of 30 feet per minute. If the tip of the rod is 10 feet above the water and the trout is at the surface of the water, how fast is the trout being pulled toward the angler? (Suggestion: Draw a picture and use the Pythagorean formula.)

**Example 5:** When rain is falling vertically, the amount (volume) of rain collected in a cylinder is proportional to the area of the opening of the cylinder. If you place a narrow cylindrical glass and a wide cylindrical glass out in the rain (Fig. 7),

(a) which glass will collect water faster, and

(b) in which glass will the water level rise faster?

Solution: Let's assume that the smaller glass has a radius of $r$ and the larger has a radius of $R$, $R > r$, so the areas of their openings are $\pi r^2$ and $\pi R^2$ respectively.

(a) The smaller glass will collect water at the rate $\frac{dv}{dt} = K\pi r^2$, and the larger at the rate $\frac{dV}{dt} = K\pi R^2$, so $\frac{dV}{dt} > \frac{dv}{dt}$, and the larger glass will collect water faster than the smaller glass.

(b) The volume of water in each glass is a function of the radius of the glass and the height of the water in the glass: $v = \pi r^2 h$ and $V = \pi R^2 H$ where $h$ and $H$ are the heights of the water levels in the smaller and larger glasses, respectively. The heights $h$ and $H$ vary with $t$ (are functions of $t$) so

$$\frac{dv}{dt} = \frac{d(\pi r^2 h)}{dt} = \pi r^2 \frac{dh}{dt}$$

and

$$\frac{dH}{dt} = \frac{dV/dt}{\pi R^2} = \frac{K\pi r^2}{\pi r^2} = K \quad \text{(we got } \frac{dv}{dt} = K\pi r^2 \text{ in part (a)).}$$

Similarly,

$$\frac{dV}{dt} = \frac{d(\pi R^2 H)}{dt} = \pi R^2 \frac{dH}{dt}$$

so

$$\frac{dH}{dt} = \frac{dV/dt}{\pi R^2} = \frac{K\pi R^2}{\pi R^2} = K.$$  

Then $\frac{dh}{dt} = K = \frac{dH}{dt}$ so the water level in each glass is rising at the same rate. In a one minute period, the larger glass will collect more rain, but the larger glass also requires more rain to raise its water level by each inch. How do you think the volumes and water levels would change if we placed a small glass and a large plastic box side by side in the rain?
PROBLEMS FOR SOLUTION

1. An expandable sphere is being filled with liquid at a constant rate from a tap (imagine a water balloon connected to a faucet). When the radius of the sphere is 3 inches, the radius is increasing at 2 inches per minute. How fast is the liquid coming out of the tap? \[ V = \frac{4}{3} \pi r^3 \]

2. The 12 inch base of a right triangle is growing at 3 inches per hour, and the 16 inch height is shrinking at 3 inches per hour (Fig. 8)
   (a) Is the area increasing or decreasing?
   (b) Is the perimeter increasing or decreasing?
   (c) Is the hypotenuse increasing or decreasing?

3. One hour later the right triangle in Problem 2 is 15 inches long and 13 inches high changing at the same rate as in Problem 2.
   (a) Is the area increasing or decreasing now?
   (b) Is the hypotenuse increasing or decreasing now?
   (c) Is the perimeter increasing or decreasing now?

4. A young woman and her boyfriend plan to elope, but she must rescue him from his mother who has locked him in his room. The young woman has placed a 20 foot long ladder against his house and is knocking on his window when his mother begins pulling the bottom of the ladder away from the house at a rate of 3 feet per second (Fig. 10). How fast is the top of the ladder (and the young couple) falling when the bottom of the ladder is
   (a) 12 feet from the bottom of the wall?
   (b) 16 feet from the bottom of the wall?
   (c) 19 feet from the bottom of the wall?
5. The length of a 12 foot by 8 foot rectangle is increasing at a rate of 3 feet per second and the width is decreasing at 2 feet per second (Fig. 11).

(a) How fast is the perimeter changing?
(b) How fast is the area changing?

6. A circle of radius 3 inches is inside a square with 12 inch sides (Fig. 12). How fast is the area between the circle and square changing if the radius is increasing at 4 inches per minute and the sides are increasing at 2 inches per minute?

7. An oil tanker in Puget Sound has sprung a leak, and a circular oil slick is forming (Fig. 13). The oil slick is 4 inches thick everywhere, is 100 feet in diameter, and the diameter is increasing at 12 feet per hour. Your job, as the Coast Guard commander or the tanker's captain, is to determine how fast the oil is leaking from the tanker.

8. A mathematical species of slug has a semicircular cross section and is always 5 times as long as it is high (Fig. 14). When the slug is 5 inches long, it is growing at .2 inches per week.

(a) How fast is its volume increasing?
(b) How fast is the area of its "foot" (the part of the slug in contact with the ground) increasing?

9. Lava flowing from a hole at the top of a hill is forming a conical mountain whose height is always the same as the width of its base (Fig. 15). If the mountain is increasing in height at 2 feet per hour when it is 500 feet high, how fast is the lava flowing (how fast is the volume of the mountain increasing)? \( V = \frac{1}{3} \pi r^2 h \)
10. A six foot tall person is walking away from a 14 foot tall lamp post at 3 feet per second (Fig. 16). When the person is 10 feet from the lamp post,
(a) How fast is the length of the person's shadow changing?
(b) How fast is the tip of the shadow moving away from the lamp post?

11. Answer parts (a) and (b) in Problem 10 for when the person is 20 feet from the lamp post.

12. Water is being poured at a rate of 15 cubic feet per minute into a conical reservoir which is 20 feet deep and has a top radius of 10 feet (Fig. 17).
(a) How long will it take to fill the empty reservoir?
(b) How fast is the water level rising when the water is 4 feet deep?
(c) How fast is the water level rising when the water is 16 feet deep?

13. The string of a kite is perfectly taut and always makes an angle of $35^\circ$ above horizontal (Fig. 18).
(a) If the kite flyer has let out 500 feet of string, how high is the kite?
(b) If the string is let out at a rate of 10 feet per second, how fast is the kite's height increasing?

14. A small tracking telescope is viewing a hot air balloon rise from a point 1000 meters away from a point directly under the balloon (Fig. 19).
(a) When the viewing angle is $20^\circ$, it is increasing at a rate of $3^\circ$ per minute. How high is the balloon, and how fast is it rising?
(b) When the viewing angle is $80^\circ$, it is increasing at a rate of $2^\circ$ per minute. How high is the balloon, and how fast is it rising?

15. The 8 foot diameter of a spherical gas bubble is increasing at 2 feet per hour, and the 12 foot long edges of a cube containing the bubble are increasing at 3 feet per hour. Is the volume contained between the spherical bubble and the cube increasing or decreasing? At what rate?
16. In general, the strength $S$ of an animal is proportional to the cross-sectional area of its muscles, and this area is proportional to the square of its height $H$, so the strength $S = aH^2$. Similarly, the weight $W$ of the animal is proportional to the cube of its height, so $W = bH^3$. Finally, the relative strength $R$ of an animal is the ratio of its strength to its weight. As the animal grows, show that its strength and weight increase, but that the relative strength decreases.

17. The snow in a hemispherical pile melts at a rate proportional to its exposed surface area (the surface area of the hemisphere). Show that the height of the snow pile is decreasing at a constant rate.

18. If the rate at which water vapor condenses onto a spherical raindrop is proportional to the surface area of the raindrop, show that the radius of the raindrop will increase at a constant rate.

19. Define $A(x)$ to be the area bounded by the $x$ and $y$ axes, the horizontal line $y = 5$, and a vertical line at $x$ (Fig. 20).

(a) Find a formula for $A$ as a function of $x$.

(b) Determine \( \frac{dA(x)}{dx} \) when $x = 1$, 2, 4 and 9.

(c) Suppose $x$ is a function of time, $x(t) = t^2$, and find a formula for $A$ as a function of $t$.

(d) Determine \( \frac{dA}{dt} \) when $t = 1$, 2, and 3.

(e) Suppose $x(t) = 2 + \sin(t)$. Find a formula for $A(t)$ and determine \( \frac{dA}{dt} \).

20. The point $P$ is going around the circle $x^2 + y^2 = 1$ twice a minute (Fig. 21). How fast is the distance between the point $P$ and the point $(4,3)$ changing

(a) when $P = (1,0)$?  (b) when $P = (0,1)$?  (c) when $P = (.8, .6)$?

(Suggestion: Write $x$ and $y$ as parametric functions of time $t$.)
21. You are walking along a sidewalk toward a 40 foot wide sign which is adjacent to the sidewalk and perpendicular to it (Fig. 22).

(a) If your viewing angle \( \theta \) is 10\( ^\circ \), then how far are you from the nearest corner of the sign?

(b) If your viewing angle is 10\( ^\circ \) and you are walking at 25 feet per minute, then how fast is your viewing angle changing?

(c) If your viewing angle is 10\( ^\circ \) and is increasing at 2\( ^\circ \) per minute, then how fast are you walking?

Section 3.7  

PRACTICE Answers

Practice 1: The surface area of the cylinder is \( SA = 2\pi rh + 2\pi r^2 \). From the Example, we know that \( h' = 7 \) m/s and \( r' = 3 \) m/s, and we want to know how fast the surface area is changing when \( h = 5 \) m and \( r = 6 \) m.

\[
\frac{dSA}{dt} = 2\pi rh' + 2\pi r'h + 2\pi r^2r'
\]

\[
= 2\pi(6 \text{ m})(7 \text{ m/s}) + 2\pi(3 \text{ m/s})(5 \text{ m}) + 2\pi(2\cdot6 \text{ m})(3 \text{ m/s}) = 186\pi \text{ m}^2/\text{s}
\]

\[
\approx 584.34 \text{ square meters per second}. \quad (\text{Note that the units represent a rate of change of area.)}
\]

Practice 2: The volume of the cylinder is \( V = \text{(area of the bottom)(height)} = \pi r^2h \). We are told that \( r' = -3 \) m/s, and that \( h = 5 \) m, \( r = 6 \) m, and \( h' = 7 \) m/s.

\[
\frac{dV}{dt} = \pi r^2h' + \pi 2r^2r'h = \pi(6 \text{ m})^2(7 \text{ m/s}) + \pi(2\cdot6 \text{ m})(-3 \text{ m/s})(5 \text{ m}) = 72\pi \text{ m}^3/\text{s}
\]

\[
\approx 226.19 \text{ cubic meters per second}. \quad (\text{Note that the units represent a rate of change of volume.)}
\]
**Practice 3:**  Fig. 23 represents the situation described in this problem. We are told that \( L' = -30 \text{ ft/min} \). The variable \( F \) represents the distance of the fish from the angler, and we are asked to find \( F' \), the rate of change of \( F \) when \( L = 60 \text{ ft} \).

Fortunately, the problem contains a right triangle so there is a formula (the Pythagorean formula) connecting \( F \) and \( L \): \( F^2 + 10^2 = L^2 \) so

\[
F = \sqrt{L^2 - 100}.
\]

Then

\[
F' = \frac{1}{2} (L^2 - 100)^{-1/2} \frac{d(L^2 - 100)}{dt} = \frac{2L \cdot L'}{2 \sqrt{L^2 - 100}}.
\]

When \( L = 60 \text{ feet} \), \( F' = \frac{2(60 \text{ ft})(-30 \text{ ft/min})}{2 \sqrt{(60 \text{ ft})^2 - (10 \text{ ft})^2}} \approx \frac{-3600 \text{ ft}^2/\text{min}}{118.32 \text{ ft}} = -30.43 \text{ ft/min} \).

We could also find \( F' \) implicitly: \( F^2 = L^2 - 100 \) so, differentiating each side,

\[
2F \cdot F' = 2L \cdot L' \quad \text{and} \quad F' = \frac{L \cdot L'}{F}.
\]

Then we could use the given values for \( L \) and \( L' \) and value of \( F \) (found using the Pythagorean formula) to evaluate \( F' \).