Chapter 10: Acceleration, Vibration and Shock Measurement  
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1. Introduction
This chapter examines the measurement of acceleration, vibration and shock. It starts by modelling the basic structure at the heart of any accelerometer: a moving mass connected to a damper and spring.

2. Modelling of the Seismic Accelerometer
We start by modelling the response of the basic so-called Seismic accelerometer, which consists of mass that is connected to the frame by a spring and damper. This forms a second order system. When the case is subjected to an acceleration \( a \), the mass will move in the opposite direction. The system is shown in Figure 1 below.

![Figure 1: Seismic accelerometer.](image)

When an external acceleration is applied to casing of the accelerometer and in the absence of no damping or spring force, the mass will accelerate in the opposite direction to the external applied force to the value of \( a \). We can think of the part of the external force that is affecting the mass \( m \) is \( m \cdot a \). But in the presence of the spring force and damper force, this force is reduced by an amount equivalent to the damper force and the spring force.

\[
m \cdot a - k_s \cdot x - k_d \cdot \dot{x} = m \cdot x
\]

Rearranging gives:

\[
m \cdot a = m \cdot x + k_d \cdot \dot{x} + k_s \cdot x
\]

Taking the Laplace transform gives:
\[ m \cdot A(s) = m \cdot X(s) \cdot s^2 + k_d \cdot X(s) \cdot s + k_s \cdot X(s) \]

\[ \frac{X(s)}{A(s)} = \frac{m}{m \cdot s^2 + k_d \cdot s + k_s} = \frac{m}{k_s} \cdot \frac{1}{s^2 + \frac{k_d}{k_s} \cdot s + 1} \]

We note from this that the output of the system is displacement and the input is acceleration. Thus the sensitivity of this block is \( m \cdot \text{m}^{-1} \cdot \text{s}^2 \) which simplifies to \( \text{s}^2 \). All accelerometers convert the original acceleration to displacement and then the displacement to an electrical quantity (voltage or current). In fact accelerometers differ from each other by the method in which they convert displacement to the electrical quantity. If the output of the accelerometer is voltage, then the sensitivity of the system will have units of \( \text{V} \cdot \text{m}^{-1} \cdot \text{s}^2 \) (or more practically \( \text{mV} \cdot \text{m}^{-1} \cdot \text{s}^2 \)).

From the equation above, it can be seen that the steady state gain of the system is:

\[ K = \frac{m}{k_s} \]

The natural undamped frequency can be found as follows:

\[ \frac{1}{\omega_n^2} = \frac{m}{k_s} \Rightarrow \omega_n = \sqrt{\frac{k_s}{m}} \]

The damping ratio can be found as follows:

\[ \frac{2 \cdot \zeta}{\omega_n} = \frac{k_d}{k_s} \Rightarrow \zeta = \frac{k_d}{2 \cdot \sqrt{m \cdot k_s}} \]

In practice, the damping inside an accelerometer could be implemented using a viscous fluid in which the mass is suspended. The spring could be implemented using a diaphragm.

The damping is necessary for the system, otherwise once the system is excited by an input it will never stop and oscillation will continue. The optimum value for the damping constant that minimise settling time is 0.7.

3. **Closed loop version of the Seismic Accelerometer**

It is possible to implement a closed loop feedback version of the accelerometer discussed in the last section [3]. This has the advantage of improving the dynamic response, increasing accuracy and reducing non-linearities. In order for any form of closed loop negative feedback to be possible, it is necessary to have a means by which the electrical signal can be re-converted back again to the physical signal that is being measured (voltage to force or acceleration in this case).

This is shown in Figure 2 below. The displacement of the mass is detected by a potentiometer that has a transfer function of \( K_p \). This signal is then amplified via an amplifier (with gain \( K_a \) with units of \( \text{A} \cdot \text{V}^{-1} \)) and converted...
to a current that is then applied to an electromagnetic actuator that opposes the original displacement in the mass (to achieve negative feedback).

Exercise 1
Draw the new block diagram of the system above. Then derive the new transfer function of the system.

4. Accelerometer Parameters
There are a number of important parameters that need to be taken into consideration when selecting an accelerometer. These are:

1. Number of axes: Accelerometers can be uni-axial, bi-axial or tri-axial. Each of the axes will be perpendicular to the others. The axes are usually referred to as $x$, $y$ and $z$, whereby the $z$ axis is usually the vertical direction.

2. Range: This is the effective range that the accelerometer can measure and is usually expressed in multiples of g (e.g., 3 g, 50 g, 700 g...).

3. Sensitivity: This usually expressed in units of mV·m$^{-1}$·s$^2$.
4. Cross sensitivity: If an acceleration signal is applied in a direction that is perpendicular to the direction of sensitivity of the accelerometer, it should show no response. In practice however, a real life accelerometer will show some response to a signal that is perpendicular to its axis. This is termed cross sensitivity, and is a dimensionless unit, usually expressed as a percentage (ranging from 1% to 5%).

5. Resonant frequency: This is the resonant frequency of the accelerometer, usually expressed in Hz. The use must ensure that the frequency of the acceleration being measured is far from this resonant frequency.

6. Time constant: This represents the speed of response of the device.

7. Environment effects (e.g., temperature effects on viscosity of containing fluid).

8. Type of output (ratio-metric or absolute): The output of an accelerometer can either be ratio-metric or absolute. A ratio-metric output means that the output signal has to be expressed as the ratio of the output voltage divided by the power supply voltage. An absolute output is independent of the value of the power supply voltage.

5. Types of accelerometers
All accelerometers rely on the principle of converting the acceleration to a displacement and then measuring the displacement using a number of different methods. This section discusses the various types of accelerometers, mainly based on data from [1].

Any accelerometer can be classified based on the following:

1. Type of spring element.
3. Method of displacement detection.

5.1 Resistive
This type is suitable for slowly varying accelerations and low frequency vibrations in the range of 0 to 50 g. It has a resolution of 1 to 400; a cross sensitivity of ±1%; inaccuracy of around ±1%; a life expectancy of around 2 million reversals. It has a typical weight of 500 gm and a typical size of 125 cm$^3$.

5.2 Strain gauge/Piezoresistive
The sensor used within these types of devices serves as a spring element and a mass. They have a typical range of around 200 g; a resolution of around 1:1000; an inaccuracy of ±1%; cross sensitivity of ±2%. They are generally of smaller size and weight than the resistive types weighing around 25 gm at a size of around 3 cm$^3$.
5.3 LVDT
The LVDT in general has the advantage over resistive potentiometric device that it does not offer any resistance to movement and that there is no contact. LVDT devices have a range of 700 g. They suffer from an inaccuracy of ±1% of full scale. They have size similar to the resistive type (125 cm³) but are lighter at around 100 gm.

5.4 Variable inductance/variable capacitance
These devices have a range of around 40 g. They suffer an inaccuracy of ±0.25% of full scale, have a resolution of 1:10 000, and a cross sensitivity of 0.5%. Their physical size and weight are similar to the resistive devices (125 cm³ at a weight of 500 gm).

5.5 Piezoelectric
The piezoelectric crystal acts as spring and damper. These devices are smaller in size at around 15 cm³ and a weight of around 50 gm. They are not suitable for slowly varying or constant accelerations (as they act as a low pass filter). They have high output impedance and the output must be measured by with a high input impedance amplifier to avoid loading, usually called a charge amplifier. Some devices contain a high impedance charge amplifier, but this needs great care as the amplifier will be subjected to high g value being housed in the same device.

Piezoelectric devices have a resolution of ±0.1%, an inaccuracy of ±1% and offer a large of measurement in one device (0.03 g to 1000 g). They are also produced in intelligent versions.

5.6 Micro-sensors
With the advent of MEMS and NEMS, accelerometers can now be micro-machined. They usually consist of a small mass mounted on a thin Silicon membrane. Displacement is measured by the use of a piezoresistor deposited on the membrane or a capacitor etched on it.

6. Vibration
Vibration is encountered in machinery (e.g., in buildings, gearboxes, motors). Vibration measurement can be used to diagnose mechanical problems. It can reach peak accelerations of up to 100 g or greater, whereby the frequency of the oscillations and the magnitude of the displacements would very randomly.

It is worth noting that when measuring vibration, high values of acceleration are encountered even at relatively low frequency oscillations. The following calculation illustrates this concept.

Suppose a cantilevered beam is oscillating at a frequency of rad·s⁻¹ with peak displacements of $X_0$. The displacement as a function of time is:

$$x(t) = X_0 \cdot \sin(\omega \cdot t)$$

The velocity as a function of time is then:

$$v(t) = \omega \cdot X_0 \cdot \cos(\omega \cdot t)$$

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The acceleration as a function of time is then:

\[ a(t) = -\omega^2 \cdot X_0 \cdot \sin(\omega \cdot t) \]

We can see that the peak acceleration value is now multiplied by the square of the frequency in radians per second. For example if a vibration signal has a frequency of 50 Hz and a peak displacement of 8 mm, the peak acceleration is:

\[ a_{\text{peak}} = (2 \cdot \pi \cdot 50)^2 \cdot 0.008 = 80.5 \text{g} \]

In general for vibration measurement, it is better to use acceleration at high frequency (as the amplitude is large) and the velocity at lower frequencies. However, this is not practical as it is difficult to mount and calibrate velocity and displacement transducers. Hence is customary to measure acceleration at all frequencies to represent vibration.

It is also important to note the loading effect that an accelerometer can have on the measured system. If the mass of the accelerometer is \( m_a \) and the mass of the system to be measured is \( m_b \), with its true acceleration \( a_b \) then the following holds:

\[ m_b \cdot a_b = a_{\text{whole}} \cdot (m_a + m_b) \]

So the measured acceleration will be smaller than the true acceleration by an error that depends on the relative masses of the accelerometer and the system, as shown below.

\[ a_{\text{whole}} = a_b \cdot \frac{m_b}{m_a + m_b} \]

Thus it is important to ensure that the mass of the accelerometer is small such that it does not affect the reading (i.e., loading effect). This is an advantage of the piezoelectric accelerometers and the micro-sensors as they have small masses. The piezoelectric transducer is the most widely used due to its small mass and good frequency response (although it is not suitable for low frequency and constant signals).

It is important to pay attention to the frequency response of the device used compared to the expected frequency of the measured signal. As a rule of thumb the lowest frequency of the accelerometer should be 0.1 of the lower frequency in the signal to be measured and its highest frequency should be 10 time larger than the highest frequency in the signal to be measured. The following is an overview of the frequency response of some of the devices discussed earlier:

- Resistive and variable inductance: 25 Hz
- LVDT and strain gauge devices: 150 Hz
7. Shock
Shock involves the measurement of high values of acceleration such as that of an object suddenly brought to rest often because of a collision. An example is where a body is dropped and hits the floor. An accelerometer is usually required that has a very high frequency response as the acceleration signal might only last for 5 ms and a large range of around 500 $g$. Piezoelectric transducers are suitable for such an application.

Consider for example an object being dropped 10 m hitting the floor and coming to a full rest within 5 ms. It will hit the floor with a velocity of 14 m$\cdot$s$^{-1}$. As it will come to rest with 5 ms, the average acceleration will be around 286 $g$!

References