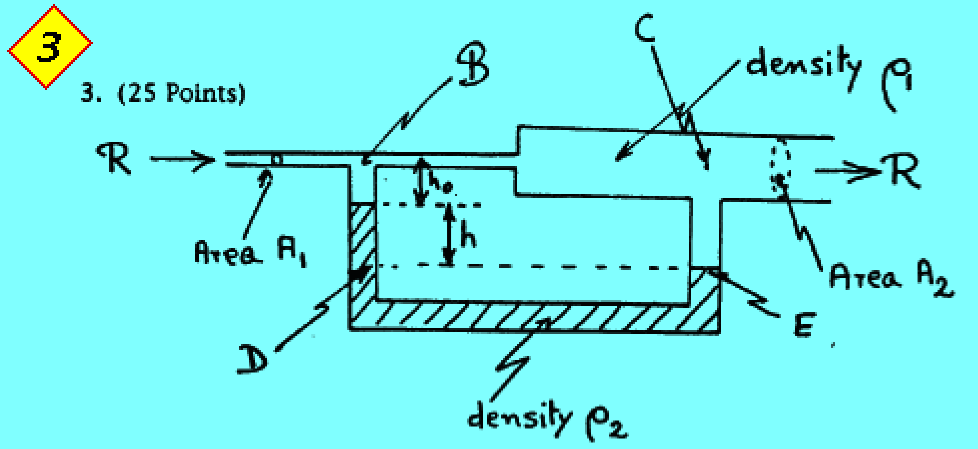
8.01 Quiz 9 Solutions, Fall 1994

3. (25 Points)



An incompressible liquid of density ρ_1 is flowing, with laminar flow, through a tube at a volume rate of R. The tube consists of two parts; the first has cross-sectional area A_1, and the second A_2. The two sections are connected by a U-tube as shown above. The U-tube is partially filled with a liquid of density ρ_2 ($\rho_2 > \rho_1$) which does not mix with the flowing liquid. The difference in height of the boundary between the two liquids on the two sides of the U-tube is h. Derive an expression for h in terms of *$R, A_1, A_2, \rho_1, \rho_2$, and g.*

Let v_1 and v_2 be the velocities of the liquid in the two tubes.

Since the flowing liquid is incompressible, by the continuity equation,

$$R = v_1 A_1 = v_2 A_2 \quad \ldots.(1)$$

By Bernoulli's equation, at the points B & C on the axis of the tube, the pressure P_1 & P_2 respectively is given by: $P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_1 v_2^2 \quad \ldots\ldots(2)$

Now at the points D & E the pressure is equal

$$\therefore \quad P_1 + \rho_1 g h_0 + \rho_2 g h = P_2 + \rho_1 g h_0 + \rho_1 g h \quad \ldots(3)$$

From (3) $h = \frac{P_2 - P_1}{g(\rho_2 - \rho_1)}$

therefore, using (1) & (2) $\quad h = \frac{R^2 \rho_1}{2g(\rho_2 - \rho_1)}\left[\frac{1}{A_1^2} - \frac{1}{A_2^2}\right]$

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