Production Functions
Xingze Wang, Ying Hsuan Lin, and Frederick Jao (2007)

14.01 Principles of Microeconomics, Fall 2007
Chia-Hui Chen
October 1, 2007

Lecture 11

Production Functions

Outline

1. Chap 6: Short Run Production Function
2. Chap 6: Long Run Production Function
3. Chap 6: Returns to Scale

1 Short Run Production Function

In the short run, the capital input is fixed, so we only need to consider the change of labor. Therefore, the production function

\[ q = F(K, L) \]

has only one variable \( L \) (see Figure 1).

**Average Product of Labor.**

\[ AP_L = \frac{\text{Output}}{\text{Labor Input}} = \frac{q}{L}. \]

Slope from the origin to \((L, q)\).

**Marginal Product of Labor.**

\[ MP_L = \frac{\partial \text{Output}}{\partial \text{Labor Input}} = \frac{\partial q}{\partial L}. \]

Additional output produced by an additional unit of labor.

Some properties about \( AP \) and \( MP \) (see Figure 2).

- When \( MP = 0 \),
  Output is maximized.
- When \( MP > AP \),
  \( AP \) is increasing.
Figure 1: Short Run Production Function.

Figure 2: Average Product of Labor and Marginal Product of Labor.

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- When $MP < AP$, $AP$ is decreasing.
- When $MP = AP$, $AP$ is maximized.

To prove this, maximize $AP$ by first order condition:

$$\frac{\partial}{\partial L} \frac{q(L)}{L} = 0$$

$$\Rightarrow$$

$$\frac{\partial q}{\partial L} \frac{1}{L} - \frac{q}{L^2} = 0$$

$$\Rightarrow$$

$$\frac{\partial q}{\partial L} = \frac{q}{L}$$

$$\Rightarrow$$

$$MP = AP.$$

**Example** (Chair Production.). Note that here $AP_L$ and $MP_L$ are not continuous, so the condition for maximizing $AP_L$ we derived above does not apply.

<table>
<thead>
<tr>
<th>Number of Workers</th>
<th>Number of Chairs Produced</th>
<th>$AP_L$</th>
<th>$MP_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Relation between Chair Production and Labor.

## 2 Long Run Production Function

Two variable inputs in long run (see Figure 3).

**Isoquants.** Curves showing all possible combinations of inputs that yield the same output (see Figure 4).

**Marginal Rate of Technical Substitution (MRTS).** Slope of Isoquants.

$$MRTS = \frac{dK}{dL}$$

How many units of $K$ can be reduced to keep $Q$ constant when we increase $L$ by one unit. Like $MRS$, we also have

$$MRTS = \frac{MP_L}{MP_k}.$$
Figure 3: Long Run Production Function.

Figure 4: $K$ vs $L$, Isoquant Curve.

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\textit{Proof.} Since \( K \) is a function of \( L \) on the isoquant curve,

\[ q(K(L), L) = 0 \]

\[ \implies \frac{\partial q}{\partial L} \frac{dK}{dL} + \frac{\partial q}{\partial L} = 0 \]

\[ \implies \frac{dK}{dL} = \frac{MP_L}{MP_K}. \]

\textbf{Perfect Substitutes (Inputs).} (see Figure 5)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{isoquant_curve}
\caption{Isoquant Curve, Perfect Substitutes.}
\end{figure}

\textbf{Perfect Complements (Inputs).} (see Figure 6)

\section{3 Returns to Scale}

Marginal Product of Capital.

\[ MP_K = \frac{\partial q(K, L)}{\partial K} \]

Marginal Product of Labor

\[ K \text{ constant} \ , \ L \uparrow \rightarrow q? \]

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OpenCourseWare (http://ocw.mit.edu), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Figure 6: Isoquant Curve, Perfect Complements.

Marginal Product of Capital

$L_{constant}, K \uparrow \rightarrow q$?

What happens to $q$ when both inputs are increased?

$K \uparrow, L \uparrow \rightarrow q$?

Increasing Returns to Scale.

- A production function is said to have increasing returns to scale if

$$Q(2K, 2L) > 2Q(K, L),$$

or

$$Q(aK, aL) = 2Q(K, L), a < 2.$$

- One big firm is more efficient than many small firms.
- Isoquants get closer as we move away from the origin (see Figure 7).
Figure 7: Isoquant Curves, Increasing Returns to Scale.