ME102: Subunit 3.3.4: Compression and Buckling

Imagine using a wooden toothpick as a small column and loading it until it bends and/or breaks. Compare in your imagination loading a short wooden dowel; it should be much easier to imagine the dowel compressing (shortening) in the axial direction without bending. We refer to the bending of the toothpick as *column buckling* and the breaking as *column fracture*. How these phenomena depend upon column dimensions, material properties, and end (or termination) conditions have been areas of substantial study. This reading summarizes those results without derivation; for further information concerning the theoretical derivation of the results presented here, I recommend the Wikipedia article "Buckling."

Columns have been characterized into three regimes: short, intermediate, and long. Short columns compress and then bulge or fracture as they are increasingly loaded. Intermediate-length columns buckle or kneel (fail at a particular point); this is called *inelastic buckling*. Whether a column is considered short, intermediate, or long depends not only on the column length, but also on its cross section and material properties; this property is often called the *slenderness ratio* and is quantitatively expressed as L_{eff}/k, where k = sqrt(I/A), I is the area moment of inertia (see subunit 2.1), and A is the cross section.

The loading force at which a long column starts to bend is called the *critical loading force*, Fcr, and can be calculated from Euler's formula given by $F_{cr} = EI\ \pi^2/L^2$, where E is the Young's modulus of the material of the column, I is the area moment of inertia of the column (see subunit 2.1), and L is the column length. This result comes about from the solution to the first bending mode of the governing second-order differential equation ($d^2w/dx^2 + (F/EI)\ w = 0$, where w is transverse deflection and x is axial position). In this equation it is necessary to use an effective column length, L_{eff}, rather than the actual column length, to accommodate the effects of boundary (or termination) conditions on the bending mode of the column. Different termination conditions—for example, rocking on a hinge or being mounted on a flat surface—influence the way in which the column buckles, and can change L_{eff} by as much as a factor of 2.

For intermediate-length columns, one convenient way of estimating the load at which column kneeling occurs involves replacing Young's modulus in Euler's formula with the tangent modulus at a particular stress value. This generally requires experimental data, and really just reflects that the column conditions are not in the linear stress-strain region of the material.

Most practical designs use considerable safety margins and only load columns to a small fraction of their critical buckling loads.