There are many ways in which an engineering structure may fail to function as intended. Fracture and catastrophic failure is one way; excessive deformation under stress is another. You briefly studied fracture of single parts in subunit 3.3.2. In the brief reading here, we will formalize limits for part (or element) stability under stress, and present an introduction to examining stresses and resulting deflections in complex structures such as trusses.

A common criterion for material failure in a part is the exceeding of the point of linearity in the elastic deformation versus the applied stress curve for the material. For ductile materials, this point corresponds to the onset of plastic (or irreversible) deformation, and for brittle materials, it corresponds to fracture. In either case, it is useful to set some upper limit for the allowable stress on the material in its function in the structure. These limits may be expressed in several slightly different ways depending on the details of the situation:

1. The maximum normal stress should be less than some limit.
2. The maximum shear stress component should be less than some limit.
3. The octahedral shear stress (or shear-stress magnitude) should be less than some limit.

In practice, stresses in complex structures may be predicted from numerical models (see subunit 2.3.1) or estimated from measurements of strain under a known load. It is common to estimate such strains in two dimensions from three or more measures of strain in three or more known laboratory directions. A coordinate transformation may then be performed to give strains in a more useful system. Once the strains are found, the corresponding stresses can be estimated from \( \sigma_i = (\varepsilon_i + \nu \varepsilon_j)E/(1-\nu^2) \) where \( \sigma_i \) is a normal stress, \( \varepsilon_i \) are strains, \( E \) is Young’s modulus, and \( \nu \) is Poisson’s ratio—for the normal stresses, and \( \tau=G\gamma \) for the shear stress.

**Truss Deflections and the Unit Virtual Load Method**

In order to estimate the displacements undergone in structures upon loading, it is useful to employ the unit virtual load method. This method is a special case of the concept of virtual work, which may be employed for several other types of problems. We present two simple examples of the method here, applied to the problem of estimating joint deflection in trusses upon loading. We do not present a derivation of the method; you may study that in future work. Rather, the examples should be sufficient for you to generalize to other problems.

First, consider a single elastic member of cross section \( A \) and length \( L \). The force exerted by the member in compression is \( F = E A \delta L/L \), where \( E \) is the elasticity (Young’s modulus) of the material, and \( \delta L \) is the deviation in length from the equilibrium value \( L \). Consider that the member is axially loaded in compression with a load \( C \). Now equate \( C \) to \( F \) in the equation above and solve for \( \delta L = -CL/EA \). This is a simple example for which we need not employ the unit virtual load formalism to calculate \( \delta L \).
but let us try it anyway to demonstrate that we obtain the same result. In this method, we apply a virtual (or fictitious) load at the point of interest (in this case, the end of the member) and in the direction of interest. We then compute the resulting loads in each of the members (in this case one). In this simple case, the fictitious load $Q = 1$ applied in compression generates a responding force ($R = -1$ for compression in the member). We then calculate the sum of $R_i T_i L_i / EA_i$ for all members in order to calculate the displacement caused by the load C (here $T$ is the load in the member and is positive for tension). In this case we have $-CL/EA$, which matches our result above.

Now let us consider a slightly more complicated example. Consider the pin-connected structure in the figure below. We wish to calculate the vertical displacement due to the presence of the vertical load $C$ at point D. The triangle is right with sides $L$, $L$, $\sqrt{2} L$. The members have cross section $A$ and Young’s modulus $E$. We will now illustrate the unit virtual load method in stepwise fashion.

1. Calculate loads in all members under the conditions of interest without any virtual loads. The load in BD is $-1.414 \, C$. The load in AD is $C$ (tension). Here we have used the methods of subunit 1.3.2.

2. Apply a unit virtual load in the direction of interest at the point of interest and calculate the loads in response to the virtual load. The results are simple here, since they just repeat those of step one: $R_{BD} = -1.414$, $R_{AD} = 1$. Note that we would apply the virtual load in the horizontal direction if we were interested in horizontal displacement.

3. Calculate the sum $R_i T_i L_i / EA_i$ for all members at the joint. In this case, it is $(2\sqrt{2} + 1) \, LC/EA = \delta_v$ (the vertical displacement of point D downward, in the same direction as the virtual load).