Bernoulli’s Equation:

\[ p_s + \frac{\rho \cdot V^2}{2} = p_t \]

Solve for Velocity:

\[ V^2 = \frac{2(p_t - p_s)}{\rho} \]
This page shows a schematic drawing of a pitot-static tube. **Pitot-Static tubes**, which are also called Prandtl tubes, are used on aircraft as speedometers. The actual tube on the aircraft is around 10 inches (25 centimeters) long with a 1/2 inch (1 centimeter) diameter. Several small holes are drilled around the outside of the tube and a center hole is drilled down the axis of the tube. The outside holes are connected to one side of a device called a **pressure transducer**. The center hole in the tube is kept separate from the outside holes and is connected to the other side of the transducer. The transducer measures the difference in pressure in the two groups of tubes by measuring the strain in a thin element using an electronic strain gauge. The pitot-static tube is mounted on the aircraft, or in a **wind tunnel**, so that the center tube is always pointed in the direction of the flow and the outside holes are perpendicular to the center tube. On some airplanes the pitot-static tube is put on a longer boom sticking out of the nose of the plane or the wing.

### Difference in Static and Total Pressure

Since the outside holes are perpendicular to the direction of flow, these tubes are pressurized by the local **random component** of the air velocity. The pressure in these tubes is the **static pressure (ps)** discussed in Bernoulli’s **equation**. The center tube, however, is pointed in the direction of travel and is pressurized by both the random and the ordered air velocity. The pressure in this tube is the **total pressure (pt)** discussed in Bernoulli’s equation. The pressure transducer measures the difference in total and static pressure which is the **dynamic pressure q**.

\[
\text{measurement} = q = pt - ps
\]

### Solve for Velocity

With the difference in pressures measured and knowing the local value of **air density** from pressure and temperature measurements, we can use Bernoulli’s equation to give us the velocity. On the graphic, the Greek symbol \( \rho \) is used for the dair density. In this text, we will use the letter \( r \). Bernoulli’s equation states that the static pressure plus one half the density times the velocity \( V \) squared is equal to the total pressure.

\[
ps + .5 * r * V^2 = pt
\]

Solving for V:
\[ V^2 = 2 \cdot \{pt - ps\} / r \]

\[ V = \sqrt{2 \cdot \{pt - ps\} / r} \]

where \textit{sqrt} denotes the square root function.

There are some practical limitations to the use of a pitot-static tube:

1. If the velocity is low, the difference in pressures is very small and hard to accurately measure with the transducer. Errors in the instrument could be greater than the measurement! So pitot-static tubes don't work very well for very low velocities.
2. If the velocity is very high (supersonic), we've violated the assumptions of Bernoulli's equation and the measurement is wrong again. At the front of the tube, a shock wave appears that will change the total pressure. There are corrections for the shock wave that can be applied to allow us to use pitot-static tubes for high speed aircraft.
3. If the tubes become clogged or pinched, the resulting pressures at the transducer are not the total and static pressures of the external flow. The transducer output is then used to calculate a velocity that is not the actual velocity of the flow. Several years ago, there were reports of icing problems occurring on airliner pitot-static probes. Output from the probes was used as part of the auto-pilot and flight control system. The solution to the icing problem was to install heaters on the probes to insure that the probe was not clogged by ice build-up.

\textit{Notice - In using this equation to determine the velocity, we must be very careful and use the proper units of measure. The air density must be specified as mass / volume (kg/m^3 or slug/ft^3) while the pressure is specified as force / area (Pa or lbs/ft^2).}
Bernoulli’s Equation

Restrictions:
- Inviscid
- Steady
- Incompressible (low velocity)
- No heat addition.
- Negligible change in height.

Along a streamline:

\[ p_s + \frac{\rho V^2}{2} = p_t \]

\[ \left( p_s + \frac{\rho V^2}{2} \right)_1 = \left( p_s + \frac{\rho V^2}{2} \right)_2 \]
In the 1700s, Daniel Bernoulli investigated the forces present in a moving fluid. This slide shows one of many forms of Bernoulli's equation. The equation appears in many physics, fluid mechanics, and airplane textbooks. The equation states that the static pressure $p_s$ in the flow plus the dynamic pressure, one half of the density $r$ times the velocity $V$ squared, is equal to a constant throughout the flow. We call this constant the total pressure $p_t$ of the flow.

As discussed on the gas properties page, there are two ways to look at a fluid; from the large, macro scale properties of the fluid that we can measure, and from the small, micro scale of the molecular motion and interaction. On this page, we will consider Bernoulli's equation from both standpoints.

**Macro Scale Derivation**

Thermodynamics is the branch of science which describes the macro scale properties of a fluid. One of the principle results of the study of thermodynamics is the conservation of energy; within a system, energy is neither created nor destroyed but may be converted from one form to another. We shall derive Bernoulli's equation by starting with the conservation of energy equation. The most general form for the conservation of energy is given on the Navier-Stokes equation page. This formula includes the effects of unsteady flows and viscous interactions. Assuming a steady, inviscid flow we have a simplified conservation of energy equation in terms of the enthalpy of the fluid:

$$h_t^2 - h_t^1 = q - wsh$$

where $h_t$ is the total enthalpy of the fluid, $q$ is the heat transfer into the fluid, and $wsh$ is the useful work done by the fluid.

**Assuming no heat transfer into the fluid, and no work done by the fluid**, we have:

$$h_t^2 = h_t^1$$

From the definition of total enthalpy:

$$e^2 + (p * v)^2 + (.5 * V^2)^2 = e^1 + (p * v)^1 + (.5 * V^2)^1$$

where $e$ is the internal energy, $p$ is the pressure, $v$ is the specific volume, and $V$ is the velocity of the fluid. From the first law of thermodynamics if there is no work and no heat transfer, the internal energy remains the same:

$$(p * v)^2 + (.5 * V^2)^2 = (p * v)^1 + (.5 * V^2)^1$$
The specific volume is the inverse of the fluid density \( r \):

\[
\frac{(p / r)2 + (.5 * V^2)2}{(p / r)1 + (.5 * V^2)1} = \text{constant}
\]

**Assuming that the flow is incompressible, the density is a constant.** Multiplying the energy equation by the constant density:

\[
(ps)2 + (.5 * r * V^2)2 = (ps)1 + (.5 * r * V^2)1 = \text{constant} = pt
\]

This is the simplest form of Bernoulli's equation and the one most often quoted in textbooks. If we make different assumptions in the derivation, we can derive other forms of the equation.

*It is important when applying any equation that you are aware of the restrictions on its use; the restrictions usually arise in the derivation of the equation when certain simplifying assumptions about the nature of the problem are made. If you ignore the restrictions, you may often get an incorrect "answer" from the equation. For instance, this form of the equation was derived while assuming that the flow was incompressible, which means that the speed of the flow is much less than the speed of sound. If you use this form for a supersonic flow, the answer will be wrong.*

**Molecular Scale Derivation**

We can make another interpretation of the equation by considering the motion of the gas molecules. The molecules within a fluid are in constant random motion and collide with each other and with the walls of an object in the fluid. The motion of the molecules gives the molecules a linear momentum and the fluid pressure is a measure of this momentum. If a gas is at rest, all of the motion of the molecules is random and the pressure that we detect is the total pressure of the gas. If the gas is set in motion or flows, some of the random components of velocity are changed in favor of the directed motion. We call the directed motion "ordered," as opposed to the disordered random motion.

We can associate a "pressure" with the momentum of the ordered motion of the gas. We call this pressure the dynamic pressure. The remaining random motion of the molecules still produces a pressure called the static pressure. At the molecular level, there is no distinction between random and ordered motion. Each molecule has a velocity in some direction until it collides with another molecule and the velocity is changed. But when you sum up all the velocities of all the molecules you will detect the ordered motion. From a conservation of energy and momentum, the static pressure plus the dynamic pressure is equal to the original total pressure in a flow (assuming we do not add or subtract energy in the flow). The form of the dynamic pressure is the density times the square of the velocity divided by two.

**Applications of Bernoulli's Equation**

The fluids problem shown on this slide is low speed flow through a tube with changing cross-sectional area. For a streamline along the center of the tube, the velocity decreases from station one to two. Bernoulli's equation describes the relation between velocity, density, and pressure for this
flow problem. Since density is a constant for a low speed problem, the equation at the bottom of the slide relates the pressure and velocity at station two to the conditions at station one.

Along a low speed airfoil, the flow is incompressible and the density remains a constant. Bernoulli's equation then reduces to a simple relation between velocity and static pressure. The surface of the airfoil is a streamline. Since the velocity varies along the streamline, Bernoulli's equation can be used to compute the change in pressure. The static pressure integrated along the entire surface of the airfoil gives the total aerodynamic force on the foil. This force can be broken down into the lift and drag of the airfoil.

Bernoulli's equation is also used on aircraft to provide a speedometer called a pitot-static tube. A pressure is quite easy to measure with a mechanical device. In a pitot-static tube, we measure the static and total pressure and can then use Bernoulli's equation to compute the velocity.