Predictor-corrector method

In mathematics, particularly numerical analysis, a **predictor-corrector method** is an algorithm that proceeds in two steps. First, the prediction step calculates a rough approximation of the desired quantity. Second, the corrector step refines the initial approximation using another means.

**Example**

In approximating the solution to a first-order ordinary differential equation, suppose one knows the solution points \( y_0 \) and \( y_t \) at times \( t_0 \) and \( t_1 \). By fitting a cubic polynomial to the points and their derivatives (obtained from the differential equation), one can predict a point \( \tilde{y}_2 \) by extrapolating to a future time \( t_2 \). Using the new value \( \tilde{y}_2 \) and its derivative there, \( \tilde{y}'_2 \) along with the previous points and their derivatives, one can then better interpolate the derivative between \( t_1 \) and \( t_2 \) to get a better approximation \( y_2 \). The interpolation and subsequent integration of the differential equation constitute the corrector step.

**Euler trapezoidal example**

Example of an Euler-trapezoidal predictor-corrector method.

In this example \( h = \Delta t \cdot t_{i+1} = t_i + \Delta t = t_i + h \)

\[ y' = f(t,y), \quad y(t_0) = y_0. \]

First calculate an initial guess value \( \tilde{y}_0 \) via Euler:

\[ \tilde{y}_0 = y_i + hf(t_i, y_i) \]

Next, improve the initial guess through iteration of the trapezoidal rule. This iteration process normally converges quickly.

\[ \tilde{y}_1 = y_i + \frac{h}{2}(f(t_i, y_i) + f(t_{i+1}, \tilde{y}_0)). \]

\[ \tilde{y}_2 = y_i + \frac{h}{2}(f(t_i, y_i) + f(t_{i+1}, \tilde{y}_1)). \]

... 

\[ \tilde{y}_n = y_i + \frac{h}{2}(f(t_i, y_i) + f(t_{i+1}, \tilde{y}_{n-1})). \]

This iteration process is repeated until some fixed value \( n \) or until the guesses converge to within some error tolerance \( e \):

\[ |\tilde{y}_n - \tilde{y}_{n-1}| \leq e \]

then use the final guess as the next step:

\[ y_{i+1} = \tilde{y}_n. \]

Note that the overall error is unrelated to convergence in the algorithm but instead to the step size and the core method, which in this example is a trapezoidal, (linear) approximation of the actual function. The step size \( h (\Delta t) \) needs to be relatively small in order to get a good approximation. See also stiff equation.
External links

- Weisstein, Eric W., "Predictor-Corrector Methods [1]" from MathWorld.

References

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