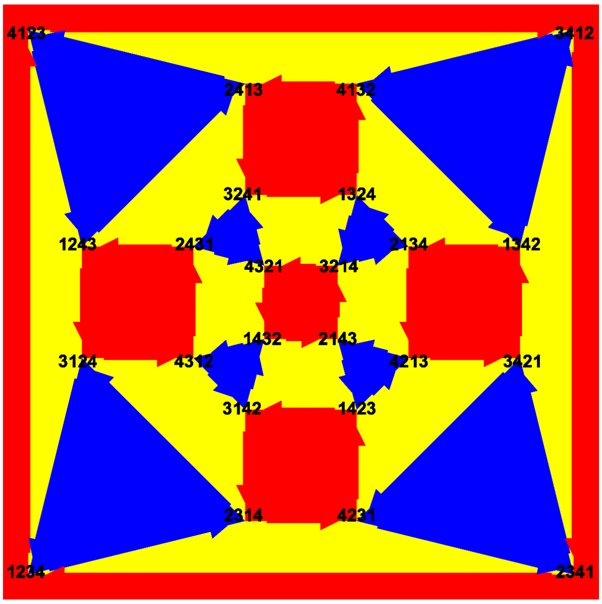
Symmetric group

In mathematics, the **symmetric group** on a set is the group consisting of all bijections of the set (all one-to-one and onto functions) from the set to itself with function composition as the group operation.[1]

The symmetric group is important to diverse areas of mathematics such as Galois theory, invariant theory, the representation theory of Lie groups, and combinatorics. Cayley's theorem states that every group G is isomorphic to a subgroup of the symmetric group on G.

This article focuses on the finite symmetric groups: their applications, their elements, their conjugacy classes, a finite presentation, their subgroups, their automorphism groups, and their representation theory. For the remainder of this article, "symmetric group" will mean a symmetric group on a finite set.



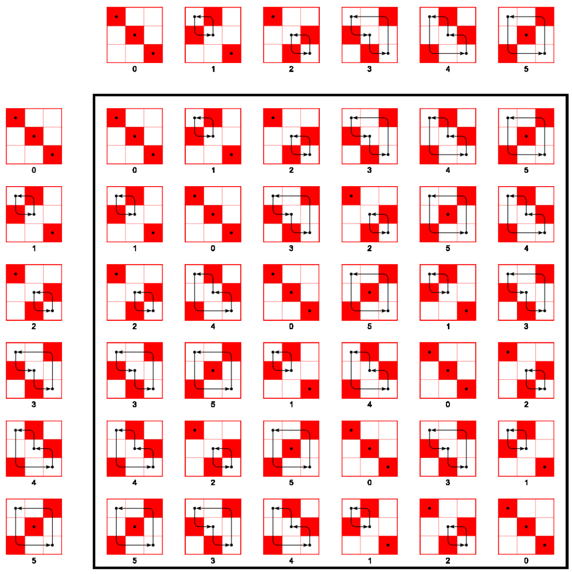
A Cayley graph of the symmetric group S_4

Definition and first properties

The **symmetric group** on a set X is the group whose underlying set is the collection of all bijections from X to X and whose group operation is that of function composition.[1] The symmetric group of **degree** n is the symmetric group on the set $X = \{ 1, 2, ..., n \}$.

The symmetric group on a set X is denoted in various ways including S_X, $\mathfrak{S}_X$, Σ_X, and Sym(X).[1] If X is the set $\{ 1, 2, ..., n \}$, then the symmetric group on X is also denoted S_n,[1] $\mathfrak{S}_n$, Σ_n, and Sym(n).

Symmetric groups on infinite sets behave quite differently than symmetric groups on finite sets, and are discussed in (Scott 1987, Ch. 11), (Dixon & Mortimer 1996, Ch. 8), and (Cameron 1999). This article concentrates on the finite symmetric groups.


Cayley table of the symmetric group S_3
(multiplication table of permutation matrices)

The symmetric group on a set of n elements has order $n!$.[2] It is abelian if and only if $n \leq 2$. For $n = 0$ and $n = 1$ (the empty set and the singleton set) the symmetric group is trivial (note that this agrees with $0! = 1! = 1$), and in these cases the alternating group equals the symmetric group, rather than being an index two subgroup. The group S_n is solvable if and only if $n \leq 4$. This is an essential part of the proof of the Abel–Ruffini theorem that shows that for every $n > 4$ there are polynomials of degree n which are not solvable by radicals, i.e., the solutions cannot be expressed by performing a finite number of operations of addition, subtraction, multiplication, division and root extraction on the polynomial's coefficients.

Applications

The symmetric group on a set of size n is the Galois group of the general polynomial of degree n and plays an important role in Galois theory. In invariant theory, the symmetric group acts on the variables of a multi-variate function, and the functions left invariant are the so-called symmetric functions. In the representation theory of Lie groups, the representation theory of the symmetric group plays a fundamental role through the ideas of Schur functors. In the theory of Coxeter groups, the symmetric group is the Coxeter group of type A_n and occurs as the Weyl group of the general linear group. In combinatorics, the symmetric groups, their elements (permutations), and their representations provide a rich source of problems involving Young tableaux, plactic monoids, and the Bruhat order. Subgroups of symmetric groups are called permutation groups and are widely studied because of their importance in understanding group actions, homogenous spaces, and automorphism groups of graphs, such as the Higman–Sims group and the Higman–Sims graph.

Elements

The elements of the symmetric group on a set X are the permutations of X.

Multiplication

The group operation in a symmetric group is function composition, denoted by the symbol $\circ$ or simply by juxtaposition of the permutations. The composition $f \circ g$ of permutations f and g, pronounced "f after g", maps any element x of X to $f(g(x))$. Concretely, let

$$f = (1\ 3)(4\ 5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$

and

$$g = (1\ 2\ 5)(3\ 4) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}.$$ (See permutation for an explanation of notation).

Applying f after g maps 1 first to 2 and then 2 to itself; 2 to 5 and then to 4; 3 to 4 and then to 5, and so on. So composing f and g gives

$$fg = f \circ g = (1\ 2\ 4)(3\ 5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}.$$

A cycle of length $L = k{\cdot}m$, taken to the k-th power, will decompose into k cycles of length m: For example ($k = 2$, $m = 3$),

$$(1\ 2\ 3\ 4\ 5\ 6)^2 = (1\ 3\ 5)(2\ 4\ 6).$$

Verification of group axioms

To check that the symmetric group on a set X is indeed a group, it is necessary to verify the group axioms of associativity, identity, and inverses. The operation of function composition is always associative. The trivial bijection that assigns each element of X to itself serves as an identity for the group. Every bijection has an inverse function that undoes its action, and thus each element of a symmetric group does have an inverse.

Transpositions

A **transposition** is a permutation which exchanges two elements and keeps all others fixed; for example (1 3) is a transposition. Every permutation can be written as a product of transpositions; for instance, the permutation g from above can be written as g = (1 5)(1 2)(3 4). Since g can be written as a product of an odd number of transpositions, it is then called an odd permutation, whereas f is an even permutation.

The representation of a permutation as a product of transpositions is not unique; however, the number of transpositions needed to represent a given permutation is either always even or always odd. There are several short proofs of the invariance of this parity of a permutation.

The product of two even permutations is even, the product of two odd permutations is even, and all other products are odd. Thus we can define the **sign** of a permutation:

$$\operatorname{sgn}(f) = \begin{cases} +1, & \text{if } f \text{ is even} \\ -1, & \text{if } f \text{ is odd.} \end{cases}$$

With this definition,

$$\operatorname{sgn} : S_n \to \{+1, -1\}$$

is a group homomorphism ({+1, −1} is a group under multiplication, where +1 is e, the neutral element). The kernel of this homomorphism, i.e. the set of all even permutations, is called the **alternating group** A_n. It is a normal subgroup of S_n, and for $n \geq 2$ it has $n! / 2$ elements. The group S_n is the semidirect product of A_n and any subgroup generated by a single transposition.

Furthermore, every permutation can be written as a product of *adjacent transpositions,* that is, transpositions of the form $(a\ a+1)$. For instance, the permutation g from above can also be written as g = (4 5)(3 4)(4 5)(1 2)(2 3)(3 4)(4 5). The representation of a permutation as a product of adjacent transpositions is also not unique.

Cycles

A cycle of *length k* is a permutation f for which there exists an element x in {1,...,n} such that $x, f(x), f^2(x), ..., f^k(x) = x$ are the only elements moved by f; it is required that $k \geq 2$ since with $k = 1$ the element x itself would not be moved either. The permutation h defined by

$$h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 3 & 5 \end{pmatrix}$$

is a cycle of length three, since $h(1) = 4$, $h(4) = 3$ and $h(3) = 1$, leaving 2 and 5 untouched. We denote such a cycle by (1 4 3). The order of a cycle is equal to its length. Cycles of length two are transpositions. Two cycles are *disjoint* if they move disjoint subsets of elements. Disjoint cycles commute, e.g. in S_6 we have (3 1 4)(2 5 6) = (2 5 6)(3 1 4). Every element of S_n can be written as a product of disjoint cycles; this representation is unique up to the order of the factors.

Special elements

Certain elements of the symmetric group of {1,2, ..., n} are of particular interest (these can be generalized to the symmetric group of any finite totally ordered set, but not to that of an unordered set).

The **order reversing permutation** is the one given by:

$$\begin{pmatrix} 1 & 2 & \cdots & n \\ n & n-1 & \cdots & 1 \end{pmatrix}.$$

This is the unique maximal element with respect to the Bruhat order and the longest element in the symmetric group with respect to generating set consisting of the adjacent transpositions (i i+1), $1 \leq i \leq n - 1$.

This is an involution, and consists of $\lfloor n/2 \rfloor$ (non-adjacent) transpositions

$$(1\,n)(2\,n-1)\cdots, \text{ or } \sum_{k=1}^{n-1} k = \frac{n(n-1)}{2} \text{ adjacent transpositions:}$$

$$(n\,n-1)(n-1\,n-2)\cdots(2\,1)(n-1\,n-2)(n-2\,n-3)\cdots,$$

so it thus has sign:

$$\operatorname{sgn}(\rho_n) = (-1)^{\lfloor n/2 \rfloor} = (-1)^{n(n-1)/2} = \begin{cases} +1 & n \equiv 0,1 \pmod{4} \\ -1 & n \equiv 2,3 \pmod{4} \end{cases}$$

which is 4-periodic in *n*.

In S_{2n}, the *perfect shuffle* is the permutation that splits the set into 2 piles and interleaves them. Its sign is also $(-1)^{\lfloor n/2 \rfloor}$.

Note that the reverse on *n* elements and perfect shuffle on 2*n* elements have the same sign; these are important to the classification of Clifford algebras, which are 8-periodic.

Conjugacy classes

The conjugacy classes of S_n correspond to the cycle structures of permutations; that is, two elements of S_n are conjugate in S_n if and only if they consist of the same number of disjoint cycles of the same lengths. For instance, in S_5, (1 2 3)(4 5) and (1 4 3)(2 5) are conjugate; (1 2 3)(4 5) and (1 2)(4 5) are not. A conjugating element of S_n can be constructed in "two line notation" by placing the "cycle notations" of the two conjugate permutations on top of one another. Continuing the previous example:

$$k = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

which can be written as the product of cycles, namely:

$$(2\ 4).$$

This permutation then relates (1 2 3)(4 5) and (1 4 3)(2 5) via conjugation, i.e.

$$(2\ 4)(1\ 2\ 3)(4\ 5)(2\ 4) = (1\ 4\ 3)(2\ 5).$$

It is clear that such a permutation is not unique.

Low degree groups

The low-degree symmetric groups have simpler structure and exceptional structure and often must be treated separately.

Sym(0) and Sym(1)

The symmetric groups on the empty set and the singleton set are trivial, which corresponds to 0! = 1! = 1. In this case the alternating group agrees with the symmetric group, rather than being an index 2 subgroup, and the sign map is trivial.

Sym(2)

The symmetric group on two points consists of exactly two elements: the identity and the permutation swapping the two points. It is a cyclic group and so abelian. In Galois theory, this corresponds to the fact that the quadratic formula gives a direct solution to the general quadratic polynomial after extracting only a single root. In invariant theory, the representation theory of the symmetric group on two points is quite simple and is seen as writing a function of two variables as a sum of its symmetric and anti-symmetric parts: Setting $f_s(x,y)$ = f(x,y) + f(y,x), and $f_a(x,y) = f(x,y) - f(y,x)$, one gets that $2 \cdot f = f_s + f_a$. This process is known as symmetrization.

Sym(3)

is isomorphic to the dihedral group of order 6, the group of reflection and rotation symmetries of an equilateral triangle, since these symmetries permute the three vertices of the triangle. Cycles of length two correspond to reflections, and cycles of length three are rotations. In Galois theory, the sign map from Sym(3) to Sym(2) corresponds to the resolving quadratic for a cubic polynomial, as discovered by Gerolamo Cardano, while the Alt(3) kernel corresponds to the use of the discrete Fourier transform of order 3 in the solution, in the form of Lagrange resolvents.

Sym(4)

The group S_4 is isomorphic to proper rotations of the cube; the isomorphism from the cube group to Sym(4) is given by the permutation action on the four diagonals of the cube. The group Alt(4) has a Klein four-group *V* as a proper normal subgroup, namely the double transpositions {(12)(34), (13)(24), (14)(23)}. This is also normal in Sym(4) with quotient Sym(3). In Galois theory, this map corresponds to the resolving cubic to a quartic polynomial, which allows the quartic to be solved by radicals, as established by Lodovico Ferrari. The Klein group can be understood in terms of the Lagrange resolvents of the quartic. The map from Sym(4) to Sym(3) also yields a 2-dimensional irreducible representation, which is an irreducible representation of a symmetric group of degree *n* of dimension below *n*−1, which only occurs for *n*=4.

Sym(5)

Sym(5) is the first non-solvable symmetric group. Along with the special linear group SL(2,5) and the icosahedral group Alt(5) × Sym(2), Sym(5) is one of the three non-solvable groups of order 120 up to isomorphism. Sym(5) is the Galois group of the general quintic equation, and the fact that Sym(5) is not a solvable group translates into the non-existence of a general formula to solve quintic polynomials by radicals. There is an exotic inclusion map $S_5 \to S_6$ as a *transitive* subgroup; the obvious inclusion map $S_n \to S_{n+1}$ fixes a point and thus is not transitive. This yields the outer automorphism of $S_6,$ discussed below, and corresponds to the resolvent sextic of a quintic.

Sym(6)

Sym(6), unlike other symmetric groups, has an outer automorphism. Using the language of Galois theory, this can also be understood in terms of Lagrange resolvents. The resolvent of a quintic is of degree 6—this corresponds to an exotic inclusion map $S_5 \to S_6$ as a *transitive* subgroup (the obvious inclusion map $S_n \to S_{n+1}$ fixes a point and thus is not transitive) and, while this map does not make the general quintic solvable, it yields the exotic outer automorphism of S_6—see automorphisms of the symmetric and alternating groups for details.

Note that while Alt(6) and Alt(7) have an exceptional Schur multiplier (a triple cover) and that these extend to triple covers of Sym(6) and Sym(7), these do not correspond to exceptional Schur multipliers of the symmetric group.

Maps between symmetric groups

Other than the trivial map $S_n \to 1 \cong S_0 \cong S_1$ and the sign map $S_n \to S_2,$ the notable maps between symmetric groups, in order of relative dimension, are:

- $S_4 \to S_3,$ corresponding to the exceptional normal subgroup $V < A_4 < S_4;$
- $S_6 \to S_6,$ (or rather, a class of such maps up to inner automorphism) corresponding to the outer automorphism of $S_6;$
- $S_5 \to S_6,$ as a transitive subgroup, yielding the outer automorphism of S_6 as discussed above.

Properties

Symmetric groups are Coxeter groups and reflection groups. They can be realized as a group of reflections with respect to hyperplanes $x_i = x_j, 1 \leq i < j \leq n$. Braid groups B_n admit symmetric groups S_n as quotient groups.

Cayley's theorem states that every group *G* is isomorphic to a subgroup of the symmetric group on the elements of *G*, as a group acts on itself faithfully by (left or right) multiplication.

Relation with alternating group

For *n*≥5, the alternating group A_n is simple, and the induced quotient is the sign map: $A_n \to S_n \to C_2$ which is split by taking a transposition of two elements. Thus S_n is the semidirect product $A_n \rtimes C_2$, and has no other proper normal subgroups, as they would intersect A_n in either the identity (and thus themselves be the identity or a 2-element group, which is not normal), or in A_n (and thus themselves be A_n or S_n).

S_n acts on its subgroup A_n by conjugation, and for $n \neq 6$, S_n is the full automorphism group of A_n : $\operatorname{Aut}(A_n) \cong S_n$. Conjugation by even elements are inner automorphisms of A_n while the outer automorphism of A_n of order 2 corresponds to conjugation by an odd element. For $n = 6$, there is an exceptional outer automorphism of A_n so S_n is not the full automorphism group of A_n.

Conversely, for $n \neq 6$, S_n has no outer automorphisms, and for $n \neq 2$ it has no center, so for $n \neq 2, 6$ it is a complete group, as discussed in automorphism group, below.

For $n \geq 5$, S_n is an almost simple group, as it lies between the simple group A_n and its group of automorphisms.

Generators and relations

The symmetric group on *n*-letters, S_n, may be described as follows. It has generators: $\sigma_1, \ldots, \sigma_{n-1}$ and relations:

- ${\sigma_i}^2 = 1,$
- $\sigma_i\sigma_j = \sigma_j\sigma_i \text{ if } j \neq i \pm 1,$
- $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}.$

One thinks of σ_i as swapping the *i*-th and *i+1*-st position.

Other popular generating sets include the set of transpositions that swap 1 and *i* for $2 \leq i \leq n$ and any set containing an n-cycle and a 2-cycle.

Subgroup structure

A subgroup of a symmetric group is called a permutation group.

Normal subgroups

The normal subgroups of the symmetric group are well understood in the finite case. The alternating group of degree *n* is the only non-identity, proper normal subgroup of the symmetric group of degree *n* except when *n* = 1, 2, or 4. In cases $n \leq 2$, then the alternating group itself is the identity, but in the case *n* = 4, there is a second non-identity, proper, normal subgroup, the Klein four group.

The normal subgroups of the symmetric groups on infinite sets include both the corresponding "alternating group" on the infinite set, as well as the subgroups indexed by infinite cardinals whose elements fix all but a certain cardinality of elements of the set. For instance, the symmetric group on a countably infinite set has a normal subgroup *S* consisting of all those permutations which fix all but finitely many elements of the set. The elements of *S* are each contained in a finite symmetric group, and so are either even or odd. The even elements of *S* form a characteristic subgroup of *S* called the alternating group, and are the only other non-identity, proper, normal subgroup of the symmetric group on a countably infinite set. For more details see (Scott 1987, Ch. 11.3) or (Dixon & Mortimer 1996, Ch. 8.1).

Maximal subgroups

The maximal subgroups of the finite symmetric groups fall into three classes: the intransitive, the imprimitive, and the primitive. The intransitive maximal subgroups are exactly those of the form Sym(*k*) × Sym(*n*–*k*) for 1 ≤ *k* < *n*/2. The imprimitive maximal subgroups are exactly those of the form Sym(*k*) wr Sym(*n*/*k*) where 2 ≤ *k* ≤ *n*/2 is a proper divisor of *n* and "wr" denotes the wreath product acting imprimitively. The primitive maximal subgroups are more difficult to identify, but with the assistance of the O'Nan–Scott theorem and the classification of finite simple groups, (Liebeck, Praeger & Saxl 1987) gave a fairly satisfactory description of the maximal subgroups of this type according to (Dixon & Mortimer 1996, p. 268).

Sylow subgroups

The Sylow subgroups of the symmetric groups are important examples of *p*-groups. They are more easily described in special cases first:

The Sylow *p*-subgroups of the symmetric group of degree *p* are just the cyclic subgroups generated by *p*-cycles. There are (*p* − 1)!/(*p* − 1) = (*p* − 2)! such subgroups simply by counting generators. The normalizer therefore has order *p*·(*p* − 1) and is known as a Frobenius group $F_{p(p-1)}$ (especially for *p* = 5), and as the affine general linear group, AGL(1, *p*).

The Sylow *p*-subgroups of the symmetric group of degree p^2 are the wreath product of two cyclic groups of order *p*. For instance, when *p* = 3, a Sylow 3-subgroup of Sym(9) is generated by *a* = (1, 4, 7)(2, 5, 8)(3, 6, 9) and the elements *x* = (1,2,3), *y* = (4, 5, 6), *z* = (7, 8, 9), and every element of the Sylow 3-subgroup has the form $a^i x^j y^k z^l$ for 0 ≤ *i*,*j*,*k*,*l* ≤ 2.

The Sylow *p*-subgroups of the symmetric group of degree p^n are sometimes denoted $W_p(n)$, and using this notation one has that $W_p(n+1)$ is the wreath product of $W_p(n)$ and $W_p(1)$.

In general, the Sylow *p*-subgroups of the symmetric group of degree *n* are a direct product of a_i copies of $W_p(i)$, where $0 \le a_i \le p - 1$ and $n = a_0 + p \cdot a_1 + \ldots + p^k \cdot a_k$.

For instance, $W_2(1) = C_2$ and $W_2(2) = D_8$, the dihedral group of order 8, and so a Sylow 2-subgroup of the symmetric group of degree 7 is generated by { (1,3)(2,4), (1,2), (3,4), (5,6) } and is isomorphic to $D_8 \times C_2$.

These calculations are attributed to (Kaloujnine 1948) and described in more detail in (Rotman 1995, p. 176). Note however that (Kerber 1971, p. 26) attributes the result to an 1844 work of Cauchy, and mentions that it is even covered in textbook form in (Netto 1882, §39–40).

Automorphism group

n	$\mathrm{Aut}(S_n)$	$\mathrm{Out}(S_n)$	$Z(S_n)$
$n \neq 2, 6$	S_n	1	1
$n = 2$	1	1	S_2
$n = 6$	$S_6 \rtimes C_2$	C_2	1

For $n \neq 2, 6$, S_n is a complete group: its center and outer automorphism group are both trivial.

For *n* = 2, the automorphism group is trivial, but S_2 is not trivial: it is isomorphic to C_2, which is abelian, and hence the center is the whole group.

For *n* = 6, it has an outer automorphism of order 2: $\mathrm{Out}(S_6) = C_2$, and the automorphism group is a semidirect product

$$\mathrm{Aut}(S_6) = S_6 \rtimes C_2.$$

In fact, for any set *X* of cardinality other than 6, every automorphism of the symmetric group on *X* is inner, a result first due to (Schreier & Ulam 1937) according to (Dixon & Mortimer 1996, p. 259).

Homology

The group homology of S_n is quite regular and stabilizes: the first homology (concretely, the abelianization) is:

$$H_1(S_n, \mathbf{Z}) = \begin{cases} 0 & n < 2 \\ \mathbf{Z}/2 & n \geq 2. \end{cases}$$

The first homology group is the abelianization, and corresponds to the sign map $S_n \to C_2$, which is the abelianization for $n \geq 2$; for $n < 2$ the symmetric group is trivial. This homology is easily computed as follows: S_n is generated by involutions (2-cycles, which have order 2), so the only non-trivial maps $S_n \to C_p$ are to C_2, and all involutions are conjugate, hence map to the same element in the abelianization (since conjugation is trivial in abelian groups). Thus the only possible maps $S_n \to C_2 \cong \{\pm 1\}$ send an involution to 1 (the trivial map) or to −1 (the sign map). One must also show that the sign map is well-defined, but assuming that, this gives the first homology of S_n.

The second homology (concretely, the Schur multiplier) is:

$$H_2(S_n, \mathbf{Z}) = \begin{cases} 0 & n < 4 \\ \mathbf{Z}/2 & n \geq 4. \end{cases}$$

This was computed in (Schur 1911), and corresponds to the double cover of the symmetric group, $2 \cdot S_n$.

Note that the exceptional low-dimensional homology of the alternating group ($H_1(A_3) \cong H_1(A_4) \cong C_3$, corresponding to non-trivial abelianization, and $H_2(A_6) \cong H_2(A_7) \cong C_6$, due to the exceptional 3-fold cover) does not change the homology of the symmetric group; the alternating group phenomena do yield symmetric group phenomena – the map $A_4 \twoheadrightarrow C_3$ extends to $S_4 \twoheadrightarrow S_3$, and the triple covers of A_6 and A_7 extend to triple covers of S_6 and S_7 – but these are not *homological* – the map $S_4 \twoheadrightarrow S_3$ does not change the abelianization of S_4, and the triple covers do not correspond to homology either.

The homology "stabilizes" in the sense of stable homotopy theory: there is an inclusion map $S_n \to S_{n+1}$, and for fixed *k*, the induced map on homology $H_k(S_n) \to H_k(S_{n+1})$ is an isomorphism for sufficiently high *n*. This is analogous to the homology of families Lie groups stabilizing.

The homology of the infinite symmetric group is computed in (Nakaoka 1961), with the cohomology algebra forming a Hopf algebra.

Representation theory

The representation theory of the symmetric group is a particular case of the representation theory of finite groups, for which a concrete and detailed theory can be obtained. This has a large area of potential applications, from symmetric function theory to problems of quantum mechanics for a number of identical particles.

The symmetric group S_n has order *n*!. Its conjugacy classes are labeled by partitions of *n*. Therefore according to the representation theory of a finite group, the number of inequivalent irreducible representations, over the complex numbers, is equal to the number of partitions of *n*. Unlike the general situation for finite groups, there is in fact a natural way to parametrize irreducible representation by the same set that parametrizes conjugacy classes, namely by partitions of *n* or equivalently Young diagrams of size *n*.

Each such irreducible representation can be realized over the integers (every permutation acting by a matrix with integer coefficients); it can be explicitly constructed by computing the Young symmetrizers acting on a space generated by the Young tableaux of shape given by the Young diagram.

Over other fields the situation can become much more complicated. If the field K has characteristic equal to zero or greater than n then by Maschke's theorem the group algebra KS_n is semisimple. In these cases the irreducible representations defined over the integers give the complete set of irreducible representations (after reduction modulo the characteristic if necessary).

However, the irreducible representations of the symmetric group are not known in arbitrary characteristic. In this context it is more usual to use the language of modules rather than representations. The representation obtained from an irreducible representation defined over the integers by reducing modulo the characteristic will not in general be irreducible. The modules so constructed are called *Specht modules*, and every irreducible does arise inside some such module. There are now fewer irreducibles, and although they can be classified they are very poorly understood. For example, even their dimensions are not known in general.

The determination of the irreducible modules for the symmetric group over an arbitrary field is widely regarded as one of the most important open problems in representation theory.

References

[1] Jacobson (2009), p. 31.

[2] Jacobson (2009), p. 32. Theorem 1.1.

- Cameron, Peter J. (1999), *Permutation Groups*, London Mathematical Society Student Texts, **45**, Cambridge University Press, ISBN 978-0-521-65378-7
- Dixon, John D.; Mortimer, Brian (1996), *Permutation groups*, Graduate Texts in Mathematics, **163**, Berlin, New York: Springer-Verlag, ISBN 978-0-387-94599-6, MR1409812
- Jacobson, Nathan (2009), *Basic algebra*, **1** (2nd ed.), Dover, ISBN 978-0-486-47189-1.
- Kaloujnine, Léo (1948), "La structure des p-groupes de Sylow des groupes symétriques finis" (http://www.numdam.org/item?id=ASENS_1948_3_65__239_0), *Annales Scientifiques de l'École Normale Supérieure. Troisième Série* **65**: 239–276, ISSN 0012-9593, MR0028834
- Kerber, Adalbert (1971), *Representations of permutation groups. I*, Lecture Notes in Mathematics, Vol. 240, **240**, Berlin, New York: Springer-Verlag, doi:10.1007/BFb0067943, MR0325752
- Liebeck, M.W.; Praeger, C.E.; Saxl, J. (1988), "On the O'Nan-Scott theorem for finite primitive permutation groups", *J. Austral. Math. Soc.* **44**: 389–396
- Nakaoka, Minoru (March 1961), "Homology of the Infinite Symmetric Group" (http://www.jstor.org/stable/1970333), *The Annals of Mathematics*, 2 (Annals of Mathematics) **73** (2): 229–257, doi:10.2307/1970333
- Netto, E. (1882) (in German), *Substitutionentheorie und ihre Anwendungen auf die Algebra.*, Leipzig. Teubner, JFM 14.0090.01
- Scott, W.R. (1987), *Group Theory*, New York: Dover Publications, pp. 45–46, ISBN 978-0-486-65377-8
- Schur, Issai (1911), "Über die Darstellung der symmetrischen und der alternierenden Gruppe durch gebrochene lineare Substitutionen", *Journal für die reine und angewandte Mathematik* **139**: 155–250
- Schreier, J.; Ulam, Stanislaw (1936), "Über die Automorphismen der Permutationsgruppe der natürlichen Zahlenfolge." (http://matwbn.icm.edu.pl/ksiazki/fm/fm28/fm28128.pdf) (in German), *Fundam. Math.* **28**: 258–260, Zbl: 0016.20301

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- Marcus du Sautoy: Symmetry, reality's riddle (http://www.ted.com/talks/marcus_du_sautoy_symmetry_reality_s_riddle.html) (video of a talk)

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