Finite group

In mathematics and abstract algebra, a **finite group** is a group whose underlying set *G* has finitely many elements. During the twentieth century, mathematicians investigated certain aspects of the theory of finite groups in great depth, especially the local theory of finite groups, and the theory of solvable groups and nilpotent groups. A complete determination of the structure of all finite groups is too much to hope for; the number of possible structures soon becomes overwhelming. However, the complete classification of the finite simple groups was achieved, meaning that the "building blocks" from which all finite groups can be built are now known, as each finite group has a composition series.

During the second half of the twentieth century, mathematicians such as Chevalley and Steinberg also increased our understanding of finite analogs of classical groups, and other related groups. One such family of groups is the family of general linear groups over finite fields. Finite groups often occur when considering symmetry of mathematical or physical objects, when those objects admit just a finite number of structure-preserving transformations. The theory of Lie groups, which may be viewed as dealing with "continuous symmetry", is strongly influenced by the associated Weyl groups. These are finite groups generated by reflections which act on a finite dimensional Euclidean space. The properties of finite groups can thus play a role in subjects such as theoretical physics and chemistry.

Number of groups of a given order

Given a positive integer n, it is not at all a routine matter to determine how many isomorphism types of groups of order n there are. Every group of prime order is cyclic, since Lagrange's theorem implies that the cyclic subgroup generated by any of its non-identity elements is the whole group. If n is the square of a prime, then there are exactly two possible isomorphism types of group of order n, both of which are abelian. If n is a higher power of a prime, then results of Graham Higman and Charles Sims give asymptotically correct estimates for the number of isomorphism types of groups of order n, and the number grows very rapidly as the power increases.

Depending on the prime factorization of n, some restrictions may be placed on the structure of groups of order n, as a consequence, for example, of results such as the Sylow theorems. For example, every group of order pq is cyclic when $q < p$ are primes with p-1 not divisible by q. For a necessary and sufficient condition, see cyclic number.

If n is squarefree, then any group of order n is solvable. A theorem of William Burnside, proved using group characters, states that every group of order n is solvable when n is divisible by fewer than three distinct primes. By the Feit–Thompson theorem, which has a long and complicated proof, every group of order n is solvable when n is odd.

For every positive integer n, most groups of order n are solvable. To see this for any particular order is usually not difficult (for example, there is, up to isomorphism, one non-solvable group and 12 solvable groups of order 60) but the proof of this for all orders uses the classification of finite simple groups. For any positive integer n there are at most two simple groups of n, and there are infinitely many positive integers n for which there are two non-isomorphic simple groups of order n.

Table of distinct groups of order *n*

Order *n*	# Groups[1]	Abelian	Non-Abelian
1	1	1	0
2	1	1	0
3	1	1	0
4	2	2	0
5	1	1	0
6	2	1	1
7	1	1	0
8	5	3	2
9	2	2	0
10	2	1	1
11	1	1	0
12	5	2	3
13	1	1	0
14	2	1	1
15	1	1	0
16	14	5	9
17	1	1	0
18	5	2	3
19	1	1	0
20	5	2	3
21	2	1	1
22	2	1	1
23	1	1	0
24	15	3	12
25	2	2	0

Notes

[1] John F. Humphreys, *A Course in Group Theory*, Oxford University Press, 1996, pp. 238-242.

External references

- Number of groups of order n (sequence A000001 (http://en.wikipedia.org/wiki/Oeis:a000001) in OEIS)
- A classifier (http://www.bluetulip.org/programs/finitegroups.html) for groups of small order

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